Large-scale three-dimensional object measurement: a practical coordinate mapping and image data-patching method

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In a practical three-dimensional (3-D) sensing system, the measurement of a large-scale object cannot be completed in only one operation. A relieflike object is generally divided into several subregions, an optical sensor positioned at each of these locations, and the shape of the whole object obtained by patching together all the 3-D data of the subregions. It is important to have accurate 3-D coordinates \( (x, y, z) \) for each subregion. We propose a new phase-to-height mapping algorithm and an accurate lateral coordinate calibration method with which to obtain the 3-D coordinates. After all the subregions are measured, it is necessary to transform the local coordinates into global world coordinates; here we present a new image data-patching method based on a flood algorithm. This method provides the optimal path along which to patch all the subregions into the shape of the entire object. We have measured and successfully patched a large sandy pool (9 m \( \times \) 5 m), and the reliability and feasibility of our method have been demonstrated by experiment. © 2001 Optical Society of America

1. Introduction

In optical three-dimensional (3-D) sensing, techniques based on structured light projection, such as phase-measuring profilometry\(^1,2\) and Fourier-transform profilometry\(^3,4\) have the advantages of high precision, high speed, and the ability to provide whole-field information. Especially in phase-measuring profilometry, phase mapping from digitized \( N \)-frame fringe data to a wrapped phase is a point-to-point operation; the reflective ratio and background do not influence calculation of the phase. So this technique becomes an important one in 3-D profilometry. With the recent availability of advanced and low-cost computers, solid-imaging devices and lasers, practical optical 3-D sensing has reached a commercial breakthrough point and is being applied in industry with increasing frequency.

One of the principal directions in the development of 3-D sensing is the accurate measurement of large and complex objects. Many researchers have investigated, and Chen et al. have presented a good review of, measurement methods in this field.\(^5\) For large-scale shape measurement, to obtain the several shapes from which a final large shape can be patched,\(^6,7\) one will require many sensors or sensor positions. The issues of how to patch these shapes together in a highly accurate manner and how to perform local and global coordinate transforms are thus important.

There are several ways to measure a 360-deg shape, among them the object rotation method,\(^8\) the imaging system transport technique, and a fixed imaging system with a multiple-camera approach. In the two last-named methods,\(^6,7\) the measurement is repeated from different angles to cover the measured object entirely, and all the local 3-D coordinates are transformed into a global coordinate system and patched together by a least-squares fit. The accuracy of a measurement system is determined by the patching method, either area-based\(^9\) or feature-based\(^10\) matching. Each of these methods has its advantages for specific applications.

A large measurement range and high accuracy are aims of future research. For a large measurement range, most shape-measuring systems trade accuracy for increasing size of the range. However, a system with both large measurement range and high accuracy is particularly desired for industrial applica-
tions. Lehmann et al.\textsuperscript{11} reported the measurement of a 4-m-wide area of a brick wall. A relielfike object is generally divided into a number of subregions, an optical sensor is positioned at each of these locations, and the whole shape is obtained by patching together all the subregions. This procedure generates another problem to be solved, namely, system errors such as lens distortion and aberrations must be overcome for accurate 3-D data to be obtained. Some pilot studies have been reported in an earlier paper.\textsuperscript{12} Here we propose a new phase-to-height mapping algorithm and an accurate lateral-coordinate calibration method, which are insensitive to most system errors, with which to get the 3-D coordinates. Moreover, a new image data-patching method based on a flood algorithm is presented in this paper. The concept of patching reliability is defined in this paper, and the patching path is determined according to the reliability of patching between neighboring subregions. With this method we can patch every subregion along an optimal path to obtain the shape of the whole object.

2. Basic Principles of and Problems with Phase-Measuring Profilometry

The optical setup of the phase-measuring profilometry is outlined in Fig. 1. When a sinusoidal pattern is projected onto a 3-D diffusing object, the distorted image can be expressed as

\[ I(x, y) = A(x, y) + B(x, y)\cos \phi(x, y), \]  

where \( A(x, y) \) is the background intensity, \( B(x, y) \) is the fringe contrast, and \( I(x, y) \) is the intensity detected by the CCD. The height of object \( h(x, y) \) is encoded in the phase \( \phi(x, y) \). By \( N = (N > 3) \) times phase shifting, phase function \( \phi(x, y) \) can be retrieved as follows:\textsuperscript{1}:

\[ \phi(x, y) = \arctan \left( \frac{\sum_{n=1}^{N} I_n(x, y)\sin(2n\pi/N)}{\sum_{n=1}^{N} I_n(x, y)\cos(2n\pi/N)} \right). \]  

Phase \( \phi(x, y) \) is wrapped into the range of its principal value, \( -\pi \rightarrow \pi \). Using an unwrapping algorithm, we automatically obtained phase distribution \( \phi(x, y) \) in its natural range. Reference phase \( \phi_r(x, y) \), which represents the phase distribution of the reference plane (plane 1 in Fig. 1), is stored in a computer as the system characteristics. The phase distribution caused by the distortion of the object is obtained as \( \phi_h(x, y) \):

\[ \phi_h(x, y) = \phi(x, y) - \phi_r(x, y). \]  

To determine the height of each point of the object requires phase-to-height mapping. Generally, the relation between phase \( \phi_h(x, y) \) and height \( h(x, y) \) can be written as\textsuperscript{13}

\[ \frac{1}{h(x, y)} = a(x, y) + b(x, y) \frac{1}{\phi_h(x, y)}, \]  

where \( h(x, y) \) is the relative height from the reference plane and \( a(x, y) \) and \( b(x, y) \) are system constants that can be obtained by calibration. If the distance between the reference plane and the camera (\( L \) in Fig. 1) is larger than the height of the object, the lateral world coordinate of each pixel in the CCD plane could be regarded as linear. So it is easy to get the lateral coordinates of an object. In a practical measurement, distance \( L \) is designed to be adjustable for differently scaled objects to permit high lateral resolution and accuracy. The object should fill the whole valid 3-D measuring volume as nearly as possible. In this case, the range of heights of the object is no longer much smaller than \( L \), and the lateral world coordinates of the CCD pixels are not a linear distribution. For example, if we move the object toward the camera, the image on the monitor will zoom in. This means that the lateral coordinate of a certain point in the CCD plane is no longer a constant but becomes a function of the point’s height. Moreover, with the increase in the camera’s visual angle, the influence of aberration becomes evident, especially in the border area of the CCD plane. In addition to the nonsinusoidal structure of the projected light, the relation of \( 1/\phi_h(x, y) \) to \( 1/h(x, y) \) cannot be considered simply a linear function. Here we present two new calibration methods to solve these problems.

3. Coordinate Mapping Method

A. Calibration of the Lateral Coordinate

As we can see from Fig. 1, plane 1 is the reference plane and plane 4 is the highest calibration plane. The distance between these two planes is \( H \); the object to be measured lies in this range. Point P in the CCD plane corresponds to point A in plane 1 and to point B in plane 4.

As a first step we place a rectangle whose width and length are known (assume that the width is \( rw \) mm and the length is \( rl \) mm) in plane 1. Each of the four vertices of the rectangle is marked by a dotted circle [Fig. 2(a)]. We snap this image into the computer and abstract the centroid of these four vertices. In this way, width \( rw_{\text{ccd1}} \) and length \( rl_{\text{ccd1}} \) (in pixels)
of the rectangle in the CCD plane can be determined. Then we place the rectangle in plane 4; its image on a monitor is shown in Fig. 2(b). By the same method we get the width \( rw_{\text{ccd}4} \) and the length \( rl_{\text{ccd}4} \) (in pixels). These four parameters are stored in the computer as system characteristics. Here we define

\[
R_x = \frac{rl}{rl_{\text{ccd}1}}, \quad R_x = \frac{rl}{rl_{\text{ccd}4}},
\]

\[
R_y = \frac{rw}{rw_{\text{ccd}1}}, \quad R_y = \frac{rw}{rw_{\text{ccd}4}}.
\]

The lateral coordinates can be calculated by linear interpolation. When we obtain height \( h(x, y) \) of point \((x, y)\) in the CCD plane, its lateral coordinates relative to the original point \( O \), which correspond to point \((x_0, y_0)\) in the CCD plane, can be written as

\[
X(x, y) = (x - x_0)R_x, \quad Y(x, y) = (y - y_0)R_y.
\]

B. Dual-Direction Nonlinear Phase-to-Height Mapping

Figure 3 shows two measured results obtained with the linear fit method; here \( H = 300 \) mm. Figure 3(a) is a plane that is placed at a height of 250 mm, and Fig. 3(b) is a plane placed at 30 mm. The root-mean squares are 1.04 and 1.05 mm, respectively. Considering the distortion and aberrations of the imaging system, we adopt a conic fit method. Then the phase-to-height mapping equation is

\[
\frac{1}{h(x, y)} = a(x, y) + b(x, y) \frac{1}{\phi_h(x, y)} + c(x, y) \frac{1}{\phi_h^2(x, y)}, \tag{8}
\]

where \( \phi_h(x, y) = \phi(x, y) - \phi_s(x, y) \). \( \phi(x, y) \) is the continuing phase distribution of the object. As shown in Fig. 1, we measure a standard plane object in each of planes 1, 2, 3, and 4. Plane 1 is the reference plane with a height of 0 mm and a continuous phase distribution of \( \phi_s(x, y) \). For planes 2, 3, and 4 we can get three functions for every point, just as in Eq. (8). The three height values are \( H/3, 2H/3, \) and \( H \), respectively. Then the three unknown factors \( a(x, y), b(x, y), \) and \( c(x, y) \) can be calculated and saved as system characteristics.

From Eq. (8), the error of height can be written as

\[
\Delta h(x, y) = \Delta p(x, y) \left[ b(x, y) \frac{h^2(x, y)}{\phi_h^2(x, y)} + c(x, y) \frac{h^2(x, y)}{\phi_h^2(x, y)} \right], \tag{9}
\]

where \( \Delta p(x, y) \) is the phase error. If height \( h(x, y) \) is very small, that is, if the object is quite close to the reference plane, \( \phi_h(x, y) = \phi(x, y) - \phi_s(x, y) \) is also small. In general, \( c(x, y) \) is not zero, so the value of
the second term in brackets in Eq. (9) is quite large. Consequently the relative error near the reference plane is much larger than at other places far from the reference plane. Figure 4(a) is the result of a measurement with nonlinear fitting. The object is a plane that is placed at a height of 250 mm; its rms is 0.42 mm. The rms is deduced and compared with the result shown in Fig. 3(a). But if we place the measured plane at a height of 30 mm, which is rather close to the reference plane, the rms is increased to 1.05 mm. The result is shown in Fig. 4(b).

To reduce the rms near the reference plane we adopt a dual-direction fitting method. Both of the phase distributions in plane 1 and plane 4 are stored in the computer as reference phase distributions. They are $\phi_r(x, y)$ and $\phi_{ir}(x, y)$, respectively. Then, with the same method as for a single direction, we make two sets of mapping parameter tables, which we term Tables 1 and 2 (not shown). The coefficients are, respectively, $a(x, y)$, $b(x, y)$, $c(x, y)$ and $ia(x, y)$, $ib(x, y)$, $ic(x, y)$. For Table 1, the reference plane is plane 1. For Table 2, the reference plane is plane 4.

Assume that the continuous phase distribution of the measured object is $\phi(x, y)$. When we map the height of a certain point $(x_p, y_p)$ in the CCD plane, first we calculate the two differences, $\text{dif}$, and $\text{dif}_{ir}$, as follows:

$$\phi_h = \phi(x_p, y_p) - \phi_r(x_p, y_p), \quad (10)$$
$$\phi_{ih} = \phi(x_p, y_p) - \phi_{ir}(x_p, y_p). \quad (11)$$

Then, if $\phi_h \geq \phi_{ih}$, we calculate the point’s height $h(x_p, y_p)$ by using the following equation:

$$h(x, y) = \left[ a(x, y) + b(x, y) \frac{1}{\phi_h(x, y)} ight]^{-1} + c(x, y) \frac{1}{\phi_{ih}(x, y)}. \quad (12)$$

Otherwise, if $\phi_h < \phi_{ih}$, we use

$$h(x, y) = H - \left[ ia(x, y) + ib(x, y) \frac{1}{\phi_{ih}(x, y)} \right]^{-1} + ic(x, y) \frac{1}{\phi_h(x, y)}. \quad (13)$$

Figure 4(c) shows the result measured with this method, in which the object is placed at a height of 30 mm. The rms is lower than 0.5 mm everywhere between plane 1 and plane 4.

4. Flood-Based Image Data-Patching Algorithm

The whole measured surface is divided into an array of $M \times N$ subregions, and there is some overlap between neighboring subregions. If the imaging system is $512 \times 512$ pixels, then there are $512 \times 512$ height data points in one subregion. As we saw in Subsection 3.A., the point’s lateral coordinate is relative to its height. So, compared with the uniform distribution of the CCD pixel coordinates, the lateral world coordinate distribution of the $512 \times 512$ points...
is nonuniform. Because the 3-D coordinates \((x, y, z)\) of all the data points in the subregion are known, we could resample the \(i\)th subregion into a new \(m_i \times n_i\) data array by linear interpolation, in which the lateral world coordinate distribution is uniform. For a different subregion, the sample pitch in the same direction \((x\) or \(y\) direction) is uniform. But \(m_i \neq m_j\) and \(n_i \neq n_j\) when \(i \neq j\). In our study, the sample pitches in both the \(x\) and the \(y\) directions are 3 mm. A flow chart of this process is shown in Fig. 5.

After the resampling, we patch the neighboring subregions together by a height-distribution cross-correlation technique. During the measurement the location of the measuring system can be controlled by a stepper-motor location system. In the \(x\) and \(y\) directions, the prearranged pitches of the location system are \(D_x\) and \(D_y\), respectively. Because of some uncertainty factors, such as moving error and shaking of the device, there are some errors between the actual and the prearranged positions of the measuring system. Assume that the error ranges are \(-e_x\ldots e_x\) and \(-e_y\ldots e_y\), respectively.

First, we select a subregion as the start position, \(O\). There are four neighboring subregions, \(A–D\), about subregion \(O\), as shown in Fig. 6(a). Then we calculate the height cross correlation between \(O\) and \(A\), \(O\) and \(B\), \(O\) and \(C\), and \(O\) and \(D\). For example, we compute the height cross correlations between \(O\) and \(B\), which are \(m_x \times n_x\) and \(m_y \times n_y\) height data arrays, respectively. We move \(B\) to overlap \(O\) point by point in the \(x\) and \(y\) directions and calculate its normal cross-correlation coefficient for every position (here \(B\) can move only in the ranges \(-e_x\ldots e_x\) and \(-e_y\ldots e_y\)). We find the maximum cross-correlation coefficient \(R_{OB}\) and its corresponding center position (assumed to be \(X_{OB}\) and \(Y_{OB}\)) of \(B\). Here \(R_{OB}\) is called the patch reliability between \(O\) and \(B\), or the patch reliability of \(B\). The other coefficients can be deduced by analogy; we obtain four maximum coefficients \((R_{OA}, R_{OB}, R_{OC}, R_{OD})\) and four center position coordinates \((X_{OA}, Y_{OA}; X_{OB}, Y_{OB}; X_{OC}, Y_{OC}; X_{OD}, Y_{OD})\). From these four coefficients we then select a maximum value again; assume that it is \(R_{OB}\). As the relative location between \(O\) and \(B\) is known, subregions \(O\) and \(B\) can be patched together. The concrete process is that the height equals the average of \(O\) and \(B\) in the overlap area and the value remains unchanged in other areas.

The following process is shown in Fig. 6(b): \(O\) and \(B\) have been patched into one area, \(OB\), and there are...
six subregions about $OB$, namely, A, C, D, which connect to O; and E, F, G, which connect to B. We calculate the maximum cross-correlation coefficients $R_{BE}$, $R_{BF}$, and $R_{BG}$ and the relative coordinates $X_{BE}$, $Y_{BE}$, $X_{BF}$, $Y_{BF}$, $X_{BG}$, $Y_{BG}$. Then we determine the maximum value among $R_{OA}$, $R_{OC}$, $R_{OD}$, $R_{BE}$, $R_{BF}$, and $R_{BG}$; assume that it is $R_{BG}$. According to the principle of reliability, the current patching path is from B to G. Because the relative coordinates between B and G are known, subregion G can be patched into OB.

Then the rest of the coefficients can be deduced by analogy. We calculate the cross-correlation coefficients between the patched subregion and the unpatched neighboring subregion and find the larger of them. The reliability of the patch is higher when the correlation coefficients are bigger, so the patching proceeds along the most reliable path. The subregions are patched from high reliability to low reliability, just as a flood flows from high to low, so we call it a flood algorithm. The whole shape can be obtained in this way. The advantages of this method are that the local measuring and patching errors cannot influence the whole shape and that the patching errors are limited to a minimum area. Another advantage is based on the fact that we can measure only the area of interest of the object and do not need to measure the whole area step by step. Certainly, all the subregions must be connected to one another, as we see from Fig. 7.

Fig. 8. 3-D shape of the original riverbed.

Fig. 9. (a) 3-D shape of the aggraded riverbed. (b) Photograph of the aggraded riverbed. (c) Gray-scale height map of the aggraded riverbed. The origin point is at the top right-hand corner. (d) Local 3-D display of the bank.
The overall patching process is summarized as follows:

1. Resample all the subregions into a uniform distributed height data array.
2. Select a subregion \(O\) as the start position and consider it a patched area.
3. Set up a queue \(Q\) to contain an index of the subregions that are to be patched.
4. Put the entire index of the unpatched subregions that neighbor the patched area into \(Q\).
5. Calculate the patch’s reliability \(R(i)\) (\(i\) is the index of the subregion) for the index in \(Q\). For a specific unpatched subregion about \(k\), if there are more than one patched subregion about \(k\), select the maximum correlation coefficient as reliability \(R(k)\) of \(k\). Quickly sort the queue from high to low according to patch reliability \(R(i)\).
6. Patch the subregion at the exit of \(Q\).
7. Repeat steps 4–6 until queue \(Q\) is empty.

5. Experiment

We measured and patched a large, sandy pool (15 m \(\times\) 6 m) by the method described above. Our measuring system uses a PULNIX TM-6AS CCD camera, and frame-grabber board DT 3152 (Total Turnkey Solutions) is for 512 \(\times\) 512 pixels. Distances \(L\) and \(D\) (Fig. 1) are 1800 and 600 mm, respectively, and the equivalent wavelength is \(\approx 48\) mm. The total calibrated range \(H\) is 300 mm. For a one-time measurement, the valid lateral range is approximately 650 mm \(\times\) 500 mm, so the system's lateral resolving power is as good as 0.1 mm. We measured several plane objects at different heights in this range. As we described in Subsection 3.B, the standard deviation (rms) is less than 0.5 mm, which is \(\approx 1\)% of the equivalent wavelength. The precision reduces slightly when the object is located out of the calibrated range.

We adapted the sandy pool to study the aggradation of a riverbed. There are four movable boards on the bottom of the pool that can be raised and lowered. The whole measured region is 9 m \(\times\) 3000 mm, divided into 22 \(\times\) 9 subregions, each measured separately. The precision of the location system is \(\approx 20\) mm in two directions, namely, \(e_x = 20\) mm and \(e_y = 20\) mm.

The aggradation experiment has two steps. First, we measure the original shape of the riverbed. That means we must take 22 \(\times\) 9 measurements and patch them together. Figure 8 shows the 3-D shape of an original riverbed. After the alluviation of the sand-laden current, the riverbed’s shape has changed. As we can see from Fig. 9(b), the direction of the current is north (top) to south (bottom), and the current flows between two banks. Here the banks have been raised artificially and the movable bottom boards have been inclined. Then we take another 22 \(\times\) 9 measurement and patch it to get the 3-D data of the aggraded riverbed and the whole shape as in Fig. 9(a). Figure 9(b) is a photograph of the aggraded riverbed, and Fig. 9(c) is its gray-scale height map. In these figures every subregion is patched smoothly, and there are no obvious joints between subregions. To show the measurement result more clearly, in Fig. 9(d) we display part of the bank on the aggraded riverbed. With the two sets of complete region 3-D data, the aggradation of the riverbed could be analyzed quantitatively. Figures 10(a) and 10(b) show the section at \(y = 5000\) mm and at \(x = 3000\) mm, respectively.

6. Conclusion

In this paper we have presented a new dual-direction nonlinear phase-to-height mapping technique and a practical algorithm for calibration of lateral coordinates. With these methods we determined accurately the 3-D coordinates of a measured object. The accuracy of lateral coordinates is 0.1 mm, and the rms of the height is less than 0.5 mm, which is \(\approx 1\)% of the equivalent wavelength. A new image data-patching method that is based on a flood model has been proposed. This method is effective for large-scale measurement and an entire shape can be assembled by patching together all the subregions along the opti-
mal path. Our measuring system has been successfully applied in the study of riverbed aggradation.

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References