Two-neuron-based non-autonomous memristive Hopfield neural network: Numerical analyses and hardware experiments

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This paper explores a two-neuron-based non-autonomous memristive Hopfield neural network (mHNN) through numerical analyses and hardware experiments. It is interested that the locus and stability of the AC equilibrium point for the mHNN change with the time evolution. Dynamical behaviors associated with the self-coupling strength of the memristive synapse are numerically investigated by bifurcation diagrams, Lyapunov exponents and phase portraits. Particularly, bursting behaviors are revealed when the order gap exists between the natural frequency and external stimulus frequency. The interesting phenomena are illustrated through phase portraits, transmitted phase portraits, and time-domain waveforms of two cases. Moreover, breadboard experimental investigations are carried out, which effectively verify the numerical simulations.

1. Introduction

As a nonvolatile resistor with frequency-dependent pinched hysteresis loops, memristor can remember the total electric charge passed through it as time going [1]. The unique ability makes memristor behaving like electronic synapse in neural networks [2–5]. Until now, numerous memristive neural networks have been reported by introducing memristors into some classical neural networks or replacing resistive synapse weights with memristive synapse weights in these neural networks, within which rich dynamical behaviors have been revealed by numerical and circuit simulations [6–12]. Only the dynamical behaviors in small world memristive neural networks have been validated by hardware experimental measurements in recent years [9]. Unlike those memristive neural networks employing memristors as mutual-connections between neurons, a new two-neuron-based non-autonomous memristor-based Hopfield neural network (mHNN) employing a memristor as a self-connection synapse is presented. Since memristors are used as the synapses of Hopfield neural network (HNN) [6–12], these memristor-based Hopfield neural networks can be written as mHNN shortly for convenient.

The HNN is a kind of classical recurrent artificial neural network, which can be used in information processing and possible engineering applications [13–19]. Thus, the dynamical behaviors including chaos [7,9], hyperchaos [8], multi-stability [10] and hidden attractors [11,20] have inspired great interest of research. Besides, studies of biological systems show that bursting behavior plays crucial role in biological activities [21,22]. In the past few years, bursting behavior has been found in various physical systems with two time scales including coupled chaotic systems [23,24], non-smooth Chua’s system [25], periodically forced system [26,27], electronic circuits [28,29], and so on. Thus, bursting behavior may occur when the order gap exists between the natural frequency and external stimulus frequency in our proposed non-autonomous mHNN. Moreover, bursting behavior appears when the oscillation switches between quiescent state (QS) and the spiking state (SP), repetitively. Therefore, at least two bifurcation routes should be involved to lead the oscillation switching from QS to SP and vice versa [28]. Several bursting behaviors such as the fold/fold and fold/Hopf busters have been revealed [26]. Fold bifurcation (FB) is a local bifurcation in which two fixed points of a dynamical system collide and annihilated each other leading the oscillation to switching from QS to SP [26]. Hopf bifurcation (HB) is a critical point where a system’s stability switches from unstable to stable state with the emergence of pure imaginary eigenvalues, and a periodic solution arises, which means an equilibrium point of the system loses its stability.

The layout of the paper is as follows. In Section 2, a simplified memristor emulator is presented and pinched hysteresis loops are numerically and experimentally validated. In Section 3, the connection topology and mathematical model of a two-neuron-based non-autonomous mHNN are described, as well as the AC equilibrium points and the changing of their stability with the time
evolution are conducted. In Section 4, dynamical behaviors associated to the self-coupling strength are numerically revealed by bifurcation diagrams, Lyapunov exponents, and phase portraits. In Section 5, bursting behaviors are explored by phase portraits, transformed phase portraits, and time-domain waveforms. In Section 6, hardware experimental measurements are performed to further verify the numerically simulated dynamical behaviors in the non-autonomous mHNN. Finally, the conclusions of this work are drawn.

2. Simplified memristor emulator

In order to explore the action mechanisms of memristive self-connection synapse weight in the non-autonomous mHNN conveniently, a simplified memristor emulator deriving from the Ref. [30] is proposed, as shown in Fig. 1. The simplified memristor emulator consists of only six discrete components including an integrator \( U_1 \) connected two resistors \( R \) and a capacitor \( C \), an analogue multiplier \( M_1 \) and a resistor \( R_a \). It is important to stress that the resistor \( R \) in parallel to the integrating capacitor \( C \) is employed to avoid DC voltage integral drift [30]. Compare to the memristor emulator proposed in [30], the simplified memristor emulator has the merits of simpler circuit realization and mathematical model.

The mathematical model of the simplified memristor emulator can be obtained by employing Kirchhoff's circuit laws and the constitutive relations of the discrete components are expressed as

\[
i = W(v_0) v = \frac{g}{C} v_0 v
\]

\[
dv_0/dt = -\frac{1}{RC} (v_0 + v)
\]

where \( v \) and \( i \) represent the voltage and current at the input port of the simplified memristor emulator, respectively. \( v_0 \) is the inner state variable and \( g \) is the gain of the multiplier \( M_1 \). For the sake of leading the memristor emulator into a HNN system conveniently, the mathematical model can be scaled into a dimensional form by

\[
\tau = t/RC, \quad a = gR/R_a
\]

Obviously, the memductance function \( W(v_0) \) has a linear form and can be expressed as

\[
W(v_0) = \alpha v_0
\]

By selecting \( g = 1 \), \( R = 10 \, \text{k} \Omega \), \( R_a = 10 \, \text{k} \Omega \) and \( C = 100 \, \text{nF} \) as typical circuit parameters, the memductance function is calculated as \( W(v_0) = \alpha v_0 \).

With the typical circuit parameters, the frequency-dependent pinched hysteresis loops of the simplified memristor emulator is numerically simulated by (1). Herein, a sinusoidal voltage source \( v = V_m \sin(2\pi f t) \) is employed, where \( V_m \) and \( f \) are the amplitude and frequency of the stimulus, respectively. When \( V_m = 3 \, \text{V} \) is fixed and \( f \) is respectively selected as 500 Hz, 1 kHz and 10 kHz, while when \( f = 1 \, \text{kHz} \) is kept and \( V_m \) is respectively selected as 2 V, 3 V and 4 V, the voltage-current relations are numerical simulated and plotted in Fig. 2(a) and (b), respectively. Fig. 2(a) shows that the hysteresis loop pinched at the origin and shrinks as the frequency increasing continuously. Whereas Fig. 2 (b) manifests that the pinched hysteresis loop is regardless of the amplitude. The simulated results explain that the simplified memristor emulator can meet the three fingerprints of memristor [31].

To verify the frequency-dependent pinched hysteresis loops, a hardware circuit on breadboard is made. All the circuit parameters are the same as those in the numerical simulations. Tektronix AFG 3102C is taken as a sinusoidal voltage source and the phase portraits in the \( v-i \) plane are captured by a Tektronix TDS 3034C digital oscilloscope in XY mode with 600 mV/div in X direction and 2.5 mA/div in Y direction. It should be noted that for better observing in hardware experimental measurements, all the output currents sensed by current probe are magnified fifty times by enwinding the measured wire around the current inductive probe with fifty turns. The experiment results shown in Fig. 3 are consistent well with those revealed by numerical simulations.

3. Two-neuron-based non-autonomous mHNN

3.1. Model description

Hopfield neural network can be described by a set of nonlinear ordinary differential equations corresponded to \( n \)-neurons [9]. The connection topology for a two-neuron-based non-autonomous mHNN is considered in our work, as shown in Fig. 4. Herein, an external stimulus input \( I(t) \) having the form of sinusoidal function \( I(t) = I_m \sin(2\pi f t) \) is employed, where \( I_m \) and \( F \) are the stimulus-associated parameters of the amplitude and frequency, respectively.
The world HNN modeled by (4) is a three-dimensional non-autonomous memristive dynamical system. The equilibrium point of the mHNN model (4) can be expressed as

\[
\begin{align*}
\bar{x}_1 &= \text{arctanh} \left( -\frac{k}{2} \bar{x}_2 + \frac{3}{2} \tanh (\bar{x}_2) \right) \\
\bar{x}_3 &= \frac{2}{2} \bar{x}_2 - \frac{3}{2} \tanh (\bar{x}_2)
\end{align*}
\]

where \( \bar{x}_2 \) is numerically solved by

\[
\text{arctanh} \left( \frac{2}{2} \bar{x}_2 - \frac{3}{2} \tanh (\bar{x}_2) \right) + k \left( \frac{2}{2} \bar{x}_2 - \frac{3}{2} \tanh (\bar{x}_2) \right)^2 + \frac{1}{2} \tanh (\bar{x}_2) + I(\tau) = 0
\]

By linearizing (4) around the AC equilibrium point, the Jacobian matrix of the two-neuron-based non-autonomous mHNN is obtained as

\[
J = \begin{bmatrix}
-1 - kx_3 \text{sech}^2 (\bar{x}_1) & 2.8 \text{sech}^2 (\bar{x}_2) & -k \tanh (\bar{x}_1) \\
-2.4 \text{sech}^2 (\bar{x}_1) & -1 + 4 \text{sech}^2 (\bar{x}_2) & 0 \\
-\text{sech}^2 (\bar{x}_1) & 0 & -1
\end{bmatrix}
\] (7)

The characteristic equation evaluated at \( S = (\bar{x}_1, \bar{x}_2, \bar{x}_3) \) can be derived as

\[
\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0
\] (8)

where

\[
\begin{align*}
a_1 &= 3 - 4 \text{sech}^2 (\bar{x}_3) + kx_5 \text{sech}^2 (\bar{x}_1) \\
a_2 &= 3 + k \left[ 2 \bar{x}_3 - \tanh (\bar{x}_1) \right] \text{sech}^2 (\bar{x}_1) - 8 \text{sech}^2 (\bar{x}_2) \\
&+ \left( 6.72 - 4kx_3 \right) \text{sech}^2 (\bar{x}_1) \text{sech}^2 (\bar{x}_2) \\
a_3 &= 1 + k \left[ \bar{x}_3 - \tanh (\bar{x}_1) \right] \text{sech}^2 (\bar{x}_1) - 4 \text{sech}^2 (\bar{x}_2) \\
&+ \left[ 4k \tanh (\bar{x}_1) + 6.72 - 4kx_3 \right] \text{sech}^2 (\bar{x}_1) \text{sech}^2 (\bar{x}_2)
\end{align*}
\] (9)

The external stimulus input \( I(\tau) \) changes in the range of \([ -I_m, I_m ]\) with the time evolution, so one can get the values of \( x_2 \) and explore the stability of AC equilibrium point for the specified range of \( I(\tau) \) numerically. \( k = 0.2 \) and \( k = 0.45 \) with \( I_m = 2 \) are taken as two cases to explore the locus and stability of the AC equilibrium point, as shown in Fig. 5.

Observed from Fig. 5(a), it can be found that the locus of the only AC equilibrium point changes with the time evolution. According to the eigenvalues calculated by (8), the stabilities of the AC equilibrium points can be explicated and divided into three types for \( k = 0.2 \) including stable node-focus (SNF), unstable saddle focus (USF), and stable node (SN). It is worth nothing that there are two Hopf bifurcation points (HBPs) having a real eigenvalue and a pair of pure imaginary eigenvalues in (8). The locus and stability of the AC equilibrium points with the time evolution are listed in Table 1. The first and second columns are the range of \( I(\tau) \) and corresponding stability of the AC equilibrium point, as well as the third and fourth columns show several AC equilibrium points.
and eigenvalues for a single value of $k$ in the corresponding range, respectively.

Whereas for $k = 0.45$, the evolutionary law of AC equilibrium point is different from that of $k = 0.2$ in a small range of $k$ in the range of $(0.5370, 0.5641)$, there are three AC equilibrium points marked as $S_1$, $S_2$ and $S_3$ for each $k$. As a whole, the number of the AC equilibrium points is changed with $k$ in the range of $[-2, 2]$, leading to the appearance of fold bifurcation points (FBPs) in Fig. 5(b), where the up-right is an enlarged view of the rectangle zone. FB is restricted by $a_1 = 0$, $a_1 > 0$, and $a_1(a_2 - a_3) > 0$, making it jumps between different AC equilibrium points. Summary of the locus and stability of AC equilibrium points with the time evolution for $k = 0.45$ is listed in Table 2.

In particular, the AC equilibrium point and its stability change with the time evolution, leading to that unstable AC equilibrium point may be attracted into a self-excited attractor in the proposed non-autonomous mHNN [32]. The existence of HBP and FBP may result in the occurrence of bursting behavior in the proposed mHNN when two time scales are involved [26].

4. Dynamics associated with $k$

With the model [4], numerical explorations for the two-neuron-based non-autonomous mHNN are performed by MATLAB ODE23 algorithm with the time-step $\Delta t = 0.01$ and initial conditions $(0, 0, 0, 0, 0, 0)$. The dynamical behaviors associated with $k$ are performed by bifurcation diagrams and corresponding Lyapunov exponents, where the Wolf's method [33] is employed to calculated the Lyapunov exponents.

When the self-coupling strength $k$ increases from 0.05 to 0.45, the bifurcation diagrams of the system variable $x_1$ are plotted in Fig. 6(a) and the first Lyapunov exponent is shown in Fig. 6(b), respectively. Fig. 6 manifests that complex dynamical behaviors are emerged in the two-neuron-based non-autonomous mHNN, including period, chaos, period-doubling bifurcation, and periodic window.

For several values of $k$, the phase portraits in the $x_1 - x_2$ plane are numerically simulated by MATLAB software, as shown in Fig. 7. The orbits are initialized by the initial conditions $(0, 0, 0, 0, 0, 0)$, which are the same as those used in bifurcation diagrams in

### Table 1

<table>
<thead>
<tr>
<th>Range of $k$</th>
<th>Stability</th>
<th>$k$ and AC equilibrium point</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[-2, -0.9952]$</td>
<td>SNF</td>
<td>$k = -1.5585: (1.2189, 1.7520, -0.8393)$</td>
<td>$-0.7036 \pm 0.4045, -1.0896$</td>
</tr>
<tr>
<td>$-0.9952$</td>
<td>HBP</td>
<td>$k = -0.9952: (1.1678, 0.9293, -0.8299)$</td>
<td>$\pm 0.4289, -1.0815$</td>
</tr>
<tr>
<td>$0.7131$</td>
<td>USF</td>
<td>$k = 0: (0, 0, 0)$</td>
<td>$1.1862, -1.4692, -1$</td>
</tr>
<tr>
<td>$(-0.9952, 0.7131)$</td>
<td>SNF</td>
<td>$k = 0.7178: (1.1388, -0.8880, 0.8140)$</td>
<td>$-0.1020 \pm 0.1692, -0.8697$</td>
</tr>
<tr>
<td>$(0.7191, 0.7781)$</td>
<td>SN</td>
<td>$k = 0.7520: (1.4128, -1.1862, 0.8881)$</td>
<td>$-0.3368, -0.6603, -0.7920$</td>
</tr>
<tr>
<td>$(0.7781, 2)$</td>
<td>SNF</td>
<td>$k = 1.1360: (1.3086, -1.6320, 0.8639)$</td>
<td>$-0.7964 \pm 0.4146, -0.8834$</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Range of $k$</th>
<th>Stability</th>
<th>$k$ and AC equilibrium point</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[-2, -1.1851]$</td>
<td>SNF</td>
<td>$k = -1.5520: (1.3565, 1.5608, -0.8756)$</td>
<td>$-0.5495 \pm 0.0405, -1.1620$</td>
</tr>
<tr>
<td>$-1.1851$</td>
<td>HBP</td>
<td>$k = -1.1851: (2.1422, 0.9777, -0.8461)$</td>
<td>$-0.4668, -1.543$</td>
</tr>
<tr>
<td>$0.5634$</td>
<td>USF</td>
<td>$k = 0.5634: (0.9057, -0.7054, 0.7919)$</td>
<td>$-0.2763, -0.6337$</td>
</tr>
<tr>
<td>$(-1.1851, 0.5370)$</td>
<td>USF</td>
<td>$k = -0.3680: (0.2799, 0.2231, -0.2728)$</td>
<td>$0.3937 \pm 0.5904, -1.0665$</td>
</tr>
<tr>
<td>$0.5370$</td>
<td>FBP</td>
<td>$k = 0.5370: (1.3224, -1.0582, 0.8674)$</td>
<td>$0, -0.7807 \pm 0.3167$</td>
</tr>
<tr>
<td>$0.5641$</td>
<td>SN</td>
<td>$k = 0.5641: (0.9438, -0.7345, 0.7370)$</td>
<td>$0, -0.1547, -0.5642$</td>
</tr>
<tr>
<td>$(0.5370, 0.5634)$</td>
<td>USF</td>
<td>$k = 0.5425: S_1 (0.7301, -0.5721, 0.6231)$</td>
<td>$0.2584 \pm 0.0973, -0.7571$</td>
</tr>
<tr>
<td>USF</td>
<td>$S_2 = -1.2317, -0.9681, 0.8431$</td>
<td>$0.1158, -0.7314 \pm 0.3140$</td>
<td></td>
</tr>
<tr>
<td>$S_3 = 1.3814, -1.1326, 0.8813$</td>
<td>SNF</td>
<td>$S_2 = -0.9027, -0.7547, 0.7488$</td>
<td>$0.1281, -0.4520 \pm 0.1047$</td>
</tr>
<tr>
<td>$S_3 = 1.4245, -1.129, 0.8905$</td>
<td>SNF</td>
<td>$S_1 = 0.5638: S_3 = (0.9217, -0.7116, 0.7267)$</td>
<td>$-0.6108, -0.0922 \pm 0.2073$</td>
</tr>
<tr>
<td>$S_3 = 1.4245, -1.129, 0.8905$</td>
<td>FBP</td>
<td>$S_2 = 0.9702 \pm 0.7547, 0.7488$</td>
<td>$0.1281, -0.4520 \pm 0.1047$</td>
</tr>
<tr>
<td>$S_3 = -1.2448, -1.2136, 0.8906$</td>
<td>SNF</td>
<td>$S_1 = 0.5640: S_2 = (0.9343, -0.7272, 0.7326)$</td>
<td>$-0.5875, -0.0548 \pm 0.1260$</td>
</tr>
<tr>
<td>$S_3 = -1.4248, -1.2136, 0.8906$</td>
<td>SNF</td>
<td>$S_1 = 0.5552, -0.7432, 0.7421$</td>
<td>$0.0796, -0.3016, -0.5211$</td>
</tr>
<tr>
<td>$S_3 = -1.4248, -1.2136, 0.8906$</td>
<td>SNF</td>
<td>$S_1 = 0.57: (1.4301, -1.2284, 0.8917)$</td>
<td>$-0.2694, -0.8247 \pm 0.3165$</td>
</tr>
</tbody>
</table>
5. Bursting behavior

With consideration of an order gap existing between the exciting frequency of external stimulus and the natural frequency of autonomous system, the proposed two-neuron-based non-autonomous mHNN can be called as generalized autonomous system (GAS) [26]. It leads to the occurrence of bursting oscillations with the alternates between QS and SP. Under the concept of GAS, the external stimulus $I(t) = I_m \sin(2\pi F t)$ in (4) can be treated as a slow-varying parameter. Due to the boundedness of sinusoidal function, the slow-varying parameter $I(t)$ changes in the range of $[-I_m, I_m]$. Herein, $I_m = 2$ which is the same as that in Section 3.

When $F = 0.0005$ is selected, an order gap exists between the external stimulus frequency and natural frequency. For $k = 0.2$ and 0.45, bursting behaviors can be explored through phase portraits, transformed phase portrait [26], and time-domain waveform, as shown in Fig. 8. The first LEs for the two cases are $LE_1 = -0.5751$ and $LE_1 = -0.5261$, which manifests that the non-autonomous mHNN runs in periodic oscillations. At the first roughly glance, the phase portrait in the $x_1 - x_3$ plane and time-domain waveform of the variable $x_2$ for the two cases are very similar, as shown in Fig. 8(a) and (b). Thus, the transformed phase portrait is employed to explicate the difference between their bifurcation mechanisms. For $k = 0.2$, only HB occurs, leading to that the mHNN keeps the repetitive spiking oscillation and quiescent oscillation. The trajectories are dense in the dot-line rectangle due to that the spiking oscillation is always around the AC equilibrium points, as illustrated in Fig. 8(c1). Whereas for $k = 0.45$, the existence of HB and FB leads to that the trajectories not only are around the AC equilibrium points, but also transmit and deviate the AC equilibrium points.
The trajectories are dense in the up and down dot-line rectangle, as shown in Fig. 8(c2), where an enlarged view of the crowded part for the one HBP and two FBPs are illustrated. The asymmetric Hopf/ Hopf bursting and fold/Hopf bursting mechanisms are explored in the proposed non-autonomous mHNN, which are different from those symmetric ones revealing in [25,26,29].

Fig. 8. Numerically simulated phase portraits, transformed phase portraits, and time-domain waveform for different $k$: (a) phase portrait in the $x_1 - x_3$ plane; (b) time-domain waveform of variable $x_2$; (c) transformed phase portraits in the $I - x_3$ plane. (a1), (b1) and (c1) for $k = 0.2$, as well as (a2), (b2) and (c2) for $k = 0.45$.

Fig. 9. Circuit implementation: (a) Inverting hyperbolic tangent function unit circuit; (b) scheme of the two-neuron-based non-autonomous mHNN.
6. Circuit implementation and hardware experiments

The hyperbolic tangent function circuit unit is realized by a dual-transistor pair of $T_1$ and $T_2$, a module of current source $I_0$, and two operational amplifier circuits for controlling gains [9,34]. Four bipolar transistors MPS2222, two operational amplifiers TL082CP with ±15 V DC voltage sources, and eleven resistors are employed in Fig. 9(a). The circuit parameters are selected as $R = 10 \, \Omega$, $R_T = 520 \, \Omega$, $R_C = 1 \, k\Omega$, $R_T = 2 \, k\Omega$ and $R_W = 9.8 \, k\Omega$. With the specified circuit parameters, the output of the hyperbolic tangent function circuit can be deduced as

$$v_{\text{out}} = -\tanh(v_{\text{in}})$$

where $v_{\text{in}}$ and $v_{\text{out}}$ denote the input voltage and output voltage of the hyperbolic tangent function circuit unit in Fig. 9(a).

With the mathematical model (4), the two-neuron-based memristive HNN can be physically implemented by discrete circuit components, as shown in Fig. 9(b). The memristor $W$ represents

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**Fig. 10.** Experimentally captured phase portraits for different $R_1$: (a) Period-2 limit cycle at $R_1 = 125 \, k\Omega$; (b) period-4 limit cycle at $R_1 = 83.3 \, k\Omega$; (c) chaotic attractor at $R_1 = 40 \, k\Omega$; (d) period-5 limit cycle at $R_1 = 34.5 \, k\Omega$.

**Fig. 11.** Experimentally captured periodic bursting oscillation for $k = 0.2$, $f = 0.5 \, \text{Hz}$, and $A = 2 \, V$: (a) Phase portraits in the $x_1 - x_3$ plane; (b) time-domain waveform for variable $x_2$. 
the simplified memristor emulator in Fig. 1 and the two circuit modules $T_3$ and $T_6$ marked by $-\tan$ with solid box are the inverting hyperbolic tangent function circuit units drawn in Fig. 9(a).

The circuit in Fig. 9(b) has three dynamic elements of the capacitors, corresponding to three state variables of $v_1$, $v_2$, and $v_3$, respectively. Therefore, the circuit state equations for the two-neuron-based memristive HNN can be established as

$$RC_1\frac{dv_1}{dt} = -v_1 - \frac{g_{m1}}{R} v_1 \tanh(v_1) + \frac{1}{R} \tanh(v_2)$$

$$RC_2\frac{dv_2}{dt} = -v_2 - \frac{g_{m2}}{R} v_1 \tanh(v_1) + \frac{1}{R} \tanh(v_2)$$

$$RC_3\frac{dv_3}{dt} = -v_3 - \tanh(v_1)$$

(11)

Supposing that the integrating time constant $RC = 1$ ms, the resistance and the capacitance can be selected as $R = 10 \text{ k}\Omega$ and $C = 100 \text{ nF}$, respectively. The resistances are calculated as $R_1 = R_2 = 2.8 \text{ k}\Omega$, $R_3 = R_4 = 10 \text{ k}\Omega$, $R_5 = R_6 = 2.4 \text{ k}\Omega$, and $R_7 = R_8 = 2.5 \text{ k}\Omega$. For different $k$, the resistances $R_1$ can be calculated by $R_1 = R_2 a = R/k$.

The sinusoidal voltage stimulus is employed as $v(t) = A \sin(2\pi f t)$, where $A = 1$ mV and $f = f/RC$. Tektronix AFG 3102C is taken as a sinusoidal voltage stimulus. It should be addressed that one auxiliary voltage follower circuit realized by an operational amplifier AD711KN is hired to isolate the applied sinusoidal voltage source in the hardware circuit.

With the circuit schematic in Fig. 9, an analogue electronic circuit is practically set up by some commercially available components on breadboards. By tuning the resistances $R_2$, the experimentally captured attractors in the $v_1-v_2$ plane for different $k$ are shown in Fig. 10. The experimental results are captured by a Tektronix TDS 3034C digital oscilloscope in XY mode with 1 V/div in X direction and 1 V/div in Y direction. Whereas for the experimental verification of the bursting behavior, the output of Tektronix AFG 3102C is set to $A = 2$ V and $f = 0.0005/0.001$ Hz = 0.5 Hz. The Tektronix TDS 3034C digital oscilloscope works in XY mode with 500 mV/div in X direction and 250 mV/div in Y direction for better observing in bursting behavior. It is emphasized that the desired initial capacitor voltages (0 V, 0 V, 0.1 V) are difficult to assign in the hardware circuit, which are randomly sensed through turning on the hardware circuit power supplies again [30,35]. Ignoring the minor deviations caused by the calculation error, parasitic circuit parameters and active device non-idealities, the experimental results shown in Fig. 10 are well consistent with the numerical simulations in Fig. 7. Moreover, Fig. 11(a) and (b) coincides well with the numerical simulations in Fig. 8(a1) and (b1), respectively. The experimentally captured results further validate the existing dynamical behaviors in the two-neuron-based non-autonomous mHNN.

7. Conclusions

A two-neuron-based non-autonomous mHNN is presented, upon which the dynamical behaviors depending on the self-coupling strength of memristive self-connection synapse and stimulus frequency are revealed by numerical simulations and experimental measurements. Due to that the locus and stability of the AC equilibrium point change with the time evolution, complex dynamical behaviors including period, chaos, period-doubling bifurcation, and periodic window are explored. Specifically, the order gap between the natural frequency and external stimulus leads to the occurrence of bursting behavior. The presenting two-neuron-based non-autonomous mHNN has simple connection topology and feasible hardware realization, which can be taken as a paradigm for demonstrating the dynamical behavior in Hopfield neural network experimentally.

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