Determination of optimal position for both support bearing and unbalance mass of balance shaft

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Abstract
A balance shaft module is a smart device that removes harmonic vibrations of rotating machinery directly by generating an excitation with the same magnitude and an opposite phase of the vibration. However, the unbalance of a balance shaft causes a considerable bending deformation of the balance shaft as well as measurable power consumption. This paper presents an optimal conceptual design of a balance shaft by determining locations of both unbalance and supporting bearing. The optimal strategy is to minimize a normalized energy sum of both the elastic strain energy and the kinematic energy of a balance shaft. Then, an optimal design of the balance shaft is derived by the explicit formulation of the global optimum location of both the supporting bearing and the unbalance mass. The optimal design is verified with a simulation of a conceptual design of a balance shaft for a specific target engine.

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1. Introduction

Rotating machines are very common in industry and carry out effective actions or useful work by converting energy into rotation using rotating elements such as gears or chains. However, harmonic excitations of rotating elements are hard to avoid due to the nature of the operating mechanism [1–4]. The harmonic vibrations caused by a small unbalance or eccentricity of rotating elements may cause significant problems under persistent and high-speed rotation.

A balance shaft module generates a direct mechanical counterforce for a harmonic vibration, which is one of the most efficient solutions to remove such problems. The balance shaft module is an intuitive device that quenches harmonic vibrations directly by generating an excitation with the same magnitude as the vibration but with an opposite phase from unbalances on a rotor [5–16]. This approach is superior to other indirect methodologies such as an isolation of the vibration path using mounting systems [5–7,17–19] because the induced excitation will no longer be transmitted as the original vibration source is directly removed using the balance shaft module.

A creative mechanical rotating balancer, or balancing shaft, was first proposed by Lanchester [20], which consists of two oppositely rotating unbalanced rotors to reduce the cyclic engine vibration efficiently. In addition, Mewes [21] suggested the position of unbalances in the offset slider-crank mechanisms. Two classic papers contributed to develop the novel mechanism of a balance shaft in practice; however, there were few technical contents to extend for the optimal design issues of a balance shaft.

One of the well-known target systems is a vehicle engine that inherently induces the secondary harmonic excitation from a reciprocating movement of pistons [5–7]. More recently, the Lanchester-type balance shaft module has been widely used for the inline 4-cylinder vehicle engine, as shown in Fig. 1. If the equivalent reciprocal mass in Fig. 1 is given as \( m_e \) and the...
corresponding unbalances of a balance shaft is designed as \( m_{br} \) to avoid the secondary vibration from the engine, the relation between \( m_R \) and \( m_{br} \) can be formulated over the angular velocity of crankshaft, \( \omega_C \), as shown in Eq. (1) [5–8].

\[
4m_R \left( \frac{r_C}{l_p} \right) \omega_C^2 = 2 \left( m_{br} (2\omega_C)^2 \right) \\
\Rightarrow m_{br} = \frac{m_R r_C^2}{2l_p} .
\]

The structural fragility of a balance shaft module stems from the unbalance on the shaft that is an indispensible structural part for counter harmonic excitations. Such the unbalance causes not only the considerable bending deformation of the shaft but also the measurable driving torque consumption during operation. The former may cause a rub or interference between the rotor and the support bearing, and the latter decreases the fuel efficiency of the vehicle [5–7].

This paper presents an optimal conceptual design of a balance shaft by determining locations of both unbalance and supporting bearing. It begins with the formulation of an objective function that consists of the elastic strain energy and the kinematic energy of the balance shaft. Then, an optimal designed is derived with a function of both the mass ratio between symmetric and asymmetric part of the shaft and with load capacity of the support bearing. This study excludes any external force induced or transmitted by a balance shaft, so the analytical models are supposed to express a free-free condition [22–26]. In addition, the two support bearings are assumed to symmetrically located with respect to the geometric center of a balance shaft. Finally, we perform a simulation of a balance shaft for a specific target-vehicle engine and demonstrate that the derived optimal design agree well with the simulation results.

2. Optimal design formulation of balance shaft

2.1. Formulation of objective function

An optimum balance shaft design starts from the definition of objective function that should seamlessly address the two main problems such as the bending deformation of the rotating shaft and the driving torque consumption. The objective function is derived with kinetic and strain energy formula, which is explained using a simple rotor model shown in Fig. 2.
Here, the angular velocity of the rotor is denoted as $\omega_B$ with respect to the $Z$-axis and the total length of the rotor is $l_B$. The symmetric mass and its effective radius are denoted as $m_{SYM}$ and $r_{SYM}$, respectively. In addition, the asymmetric (unbalance) mass and their effective radius are $m_{ASYM}$ and $r_{ASYM}$, respectively. Accordingly, the mass moment of inertia of both symmetric and asymmetric parts are

$$I_{SYM} = m_{SYM}(3r_{SYM}^2 + l_B^2)/12$$

and

$$I_{ASYM} = m_{ASYM}(r_{ASYM}^2 + x_m^2),$$

respectively.

(1) Kinetic energy induced by mass moment of inertia

Unbalances of a balance shaft dominate the mass moment of inertia with respect to the $X$-axis and cause an additional moment on a supporting bearing. The $X$-directional moment of inertia can significantly be reduced if the support bearing is located at the radius of gyration \[27,28\]. If the radius of gyration of the rotor shown in Fig. 2 is defined as $k(x_m)$ of Eq. (2-a), the equation regarding the radius of gyration can be approximated under the assumption that $I_{SYM}$ and $l_B^2$ are much larger than $m_{ASYM}r_{ASYM}^2$ and $3r_{SYM}^2$, as shown in Eq. (2-b).

$$k(x_m) = \sqrt{\frac{I_{SYM} + m_{ASYM}(r_{ASYM}^2 + x_m^2)}{(M + m)}}, \quad (2-a)$$

$$\frac{k(x_m)}{\sqrt{I_{SYM} + m_{ASYM}(r_{ASYM}^2 + x_m^2)/r_m}} = 1 + \left[ \frac{m_{ASYM}}{m_{SYM}(3r_{SYM}^2 + l_B^2/2) + m_{ASYM}r_{ASYM}^2} \right] x_m^2 \approx 1 + \left[ \frac{12}{\gamma_m l_B^2} \right] x_m^2, \quad (2-b)$$

where, $m_{SYM}/m_{ASYM} = \gamma_m (> 1)$. Under the geometrical constraint of $x_m (0 < x_m < l_B/2)$, the radius of gyration $k(x_m)$ can be approximated as the linear formula over $x_m$ and $\gamma_m$, as shown in Eq. (3). Here, the end point fitting method is applied to two boundaries, $x_m = 0$, $l_B/2$, whose error between the Eqs. (2-b) and (3) is calculated and shown in Eq. (4). Then, the linear approximation bears about 11% error or less within feasible selection of $\gamma_m$ as illustrated in Fig. 3.
\[ k(x_m) \approx \sqrt{I_{\text{SYM}} + m_{\text{ASYM}}^2} \left( 1 + \left( \frac{1}{1 + \frac{3}{\gamma_m}} - 1 \right) x_m \right) = k_0 + k_1 x_m. \]  

(3)

\[ F_{\text{err}}(x_m) = \frac{\sqrt{1 + \left( \frac{12}{\gamma_m b^2} \right) x_m^2 - \left( \sqrt{1 + \frac{3}{\gamma_m}} - 1 \right) x_m}}{\sqrt{1 + \left( 12/\gamma_m b^2 \right) x_m^2}}. \]  

(4)

where, \( k_0 \) and \( k_1 \) are the first and second coefficients of the approximated linear line. Next, the moment of inertia with respect to \( Z \)-axis can be expressed with \( k_0 \) as shown in Eq. (5).

\[ I_Z = m_{\text{ASYM}} \left( 1 + \gamma_m \right) k_0 - \frac{\gamma_m b^2}{12}. \]  

(5)

The driving torque is highly susceptible to \( I_Z \) since the driving torque is the inner product between the mass moment of inertia and the angular velocity [27,28]. The mass moment of inertia with respect to the \( X \)-axis is, however, ignored since the angular velocity with respect to \( X \)-axis is very small due to the position of the supporting bearing at the radius of gyration. Thus, we assume that the kinematic energy induced by the mass moment of inertia is a function of only \( I_Z \), as shown in Eq. (6).

\[ T = \frac{1}{2} I_Z \omega_b^2. \]  

(6)

(2) Energy induced by bending deformation

Unbalance excitation of the rotor also causes a bending deformation of the rotor. The bending deformation of the simple rotor model in Fig. 2 can be represented with beam theories described in several classical references [29–31]. The strain energy induced by the unbalance force can be expressed as shown in Eq. (7).

\[ V = \int_0^l M(z)^2 dz. \]  

(7)

where,

\[ M(z) = \frac{m_{\text{ASYM}}^2 \omega_b^2}{E I_X} \left[ \frac{z - l_b/2 - x_m}{x_b} \cdot \frac{x_m}{x_b} \right] \left[ \frac{z + x_b - l_b/2}{x_b} \cdot \frac{x_m + x_b}{x_b} \right] \left[ \frac{z - l_b/2 - x_m}{x_b} \right]. \]

and \( E \) is Young’s modulus and \( I_X \) is the second moment of inertia about the cross section of a rotor with respect to the \( X \)-axis.
2.2. Boundary conditions for an optimal design

Both energies \( V \) and \( T \) are functions of \( x_m \) and subject to the geometry of a rotor model. The support bearing position, \( x_b \), is assumed to be located at the radius of gyration (Eq. (3)), and its inequality of boundary condition are shown in Eq. (8).

\[
x_b = k_0 + k_1 x_m, \quad 0 \leq x_b \leq \frac{l_B}{2}. \tag{8}
\]

Eq. (8) can be reformulated as the approximated linear coefficient’s formula, that leads to Eq. (9-a). Inequality (9-b) can be derived since \( I_Z \) in Eq. (5) should be positive. In addition, the relationship between \( k_0 \) and \( k_1 \) can be expressed as Eq. (9-c).

\[
0 \leq k_1 x_m + k_0 \leq \frac{l_B}{2}, \tag{9a}
\]
\[
k_0 \geq \sqrt{\frac{\gamma_m}{12(1 + \gamma_m)}} l_B, \tag{9b}
\]
\[
k_0 = \frac{l_B}{(1 + \frac{2}{\gamma_m})} k_1. \tag{9c}
\]

On the other hand, the feasible range of linear coefficients is also influenced by the load capacity of supporting bearing. If the support bearing \( (f_{b2}) \) has a limited load capacity against the full payload \( (m_{ASYM} \omega^{2} \sigma_{B}) \), the condition for limited load capacity of bearing can be expressed as Eq. (10).

\[
\frac{f_{b2}}{m_{ASYM} \omega^{2} \sigma_{B}} = \frac{x_m + k_0 + k_1 x_m}{2(k_0 + k_1 x_m)} \leq \gamma_f, \tag{10}
\]

where, \( \frac{1}{2} \leq \gamma_f \leq 1 \).

Eq. (10) can be reformulated using inequality (11) and the intersected point, denoted as # III, can be obtained when the support bearing force is the same as the maximum load capacity (equality condition of the Eq. (10)). Because the limited load capacity of the support bearing is an additional constraint, the revised feasible zone of coefficients can be represented with a line from # III to # II, as shown in Fig. 4.

\[
k_0 \geq -k_1 x_m + \frac{x_m}{2\gamma_f - 1}. \tag{11}
\]

2.3. Optimal design of balance shaft

The strategy to achieve an optimal design of a balance shaft is to minimize the total energy induced from both the moment of inertia and the bending deformation. Thus, the objective function is constructed by summing two normalized energy terms that involve the elastic strain energy \( V \) and the kinematic energy of the moment of inertia \( T \) because the direct summation of two energy terms \( V, T \) is impossible to calculate without defining the detailed design parameters, such as \( m_{SYM}, m_{ASYM}, r_{SYM} \) and \( r_{ASYM} \). With this objective function, the optimization problem is formulated to minimize the total energy subject to \( x_m (0 \leq x_m \leq l_B/2) \), as
shown in Eq. (12). Here, \( \gamma_w \) is a linear weighting factor between the two normalized energies. During the design process of a balance shaft, the detail selection of the design parameters should be accompanied after determining the location of both supporting bearing and unbalance mass, which are out of scope of this study, are excluded through the normalization of energy terms.

\[
J = \min_{x_m} \left[ \gamma_w \left( \frac{V(x_m)}{\text{norm}(V(x_m))} \right) + (1-\gamma_w) \left( \frac{T(x_m)}{\text{norm}(T(x_m))} \right) \right],
\]

where, \( 0 \leq \gamma_w \leq 1 \).

### 2.4. An heuristic approach

Local optima of each energy term are analyzed in this section. From Eq. (5), the kinematic energy of the moment of inertia (T) is proportional to the square of \( k_0 \) such that the local optimum of T should occur when \( k_0 \) is located at # I in Fig. 4. In addition, the elastic strain energy (V) can be minimized when the unbalance position is identical to that of the support bearing. Even though the length of unbalance mass (\( x_m \)) is less than the minimum length of the support bearing at # I (\( x_{b,I} \)), the best energy cancelation for V is still valid at # I since the bending moment from unbalances can be controlled by the support bearing only. The nature of each energy (TV) reveals that the objective function in Eq. (12) can be satisfied as best state whenever # I is identical to that of the support bearing.

The limited load capacity of the support bearing will not allow the complete feasible zone up to # I since the value of \( x_{b} \) can only reach to # III whenever the equality condition of the inequality (11) is satisfied, as shown in Eq. (13).

\[
k_0 + k_1 x_m = \frac{x_m}{(2\gamma_m - 1)}.
\]

When # III is identical to # I, the normalized positions of both unbalances and the support bearing are derived as \( x_m^{*} \) in Eq. (14) and \( x_{b}^{*} \) in Eq. (15), respectively, by solving the Eq. (13).

\[
x_{m}^{*} = \frac{1}{\sqrt{2\gamma_m - 1}} \left( \frac{2\gamma_m - 1}{(2\gamma_m - 1)(\sqrt{2\gamma_m - 1}) - (1 + \frac{2\gamma_m - 1}{\sqrt{2\gamma_m - 1}}) \sqrt{\gamma_m - \gamma_{m} + 1}} \right),
\]

\[
x_{b}^{*} = \frac{x_{m}^{*}}{l_b (2\gamma_f - 1)}.
\]

However, if the revised zone (III–II) becomes narrower than the previous case (I–II), or the inequality (11) is violated geometrically, the feasibility boundary of coefficients should be superseded by (III–II) instead of (I–II). In this situation, the value of \( k_0 \) will increase with the increase in the position of the unbalance mass (\( x_m \)), which leads to an increase in the kinematic energy of the moment of inertia (T). Moreover, the elastic strain energy (V) cannot keep its potential at a minimum because the position of the support bearing is always less than the position of the unbalances. Both energy conditions indicate that the local optimum over two energy variables are merged into # III, although the local optimum cannot reach to the global optimum as previously derived in Eq. (14). Therefore, we conclude that \( x_{m}^{*} \) is the unique global optima for the balance shaft over every design situations to be considered in the objective function in Eq. (12).

The best location of unbalance mass is the function of both the mass ratio, \( \gamma_m \), and the load capacity of a supporting bearing, \( \gamma_f \), and the global optimal location of the supporting bearing is also dependent on the parameters. The sensitivity of global optimum over two design parameters, \( \gamma_m \) and \( \gamma_f \) are investigated through the trace of the global optimum according to the variation of design parameters. The normalized global optima of both the unbalance mass and the supporting bearing are calculated for different values of \( \gamma_m \) and \( \gamma_f \) as illustrated in Figs. 5 and 6, respectively. Both global optima are less sensitive to \( \gamma_m \) however particularly, both values are decreasing nonlinearly as approaching to \( \gamma_m = 1 \). The global optimum of a supporting bearing is also less sensitive to \( \gamma_f \), whereas that of an unbalance mass is much sensitive. That is, the optimal location of unbalances increases rapidly as the value of \( \gamma_f \) increases. Figs. 5 and 6 show that the global optimum of an unbalance mass coincides with that of a supporting bearing whenever \( \gamma_f = 1 \) and the distance between two optimum is the only function of \( \gamma_f \) as shown in Eq. (15).

### 2.5. Numerical simulation

The conceptual design of a balance shaft was performed for the inline 4-cylinder engine manufactured by automaker A (more than 2000 CC) using the proposed optimal design strategy. The engine specifications of the target vehicle in relation to the balance shaft are listed in Table 1.
From the engine specifications in Table 1, the required unbalance \((m_{BrB})\) on a balance shaft was determined using Eq. (1), and the result is given in Eq. (16).

\[
m_{BrB} = \frac{(757.4 + 426.9) \times 49^2}{2 \times 155} = 1273.5 \text{[g cm]}.
\] (16)

Next, the length of the balance shaft \((l_B)\) is determined as well as the mass ratio \((\gamma_m)\) when considering the layout of engine module. In this simulation, we assume \(l_B = 250\) mm and \(\gamma_m = 3\). Under these conditions, we investigate the local optima of the objective function in Eq. (12) to verify the proposed optimal design strategy for two cases: the limited and the unlimited load capacity of the support bearing.

First, the local optimum of the objective function with a support bearing of unlimited load capacity is simulated according to the position of the unbalance mass \((x_m)\). The locus of local minimum of the objective function is shown in Fig. 7 and we find that the global optimum of the unbalance mass exists at a unique position, 0.2789, that is independent of the value of \(\gamma_m\), and such a position is identical to the global optimum calculated from Eq. (14) with \(\gamma_f = 1\), as shown in Eq. (17). Moreover, at the global

Fig. 5. Locus of the global optimum of both the unbalance mass and the supporting bearing over different value of the mass ratio, \(\gamma_m\). For the unbalance mass, solids (circle), \(\gamma_f = 1\); dashes (circle), \(\gamma_f = 0.8\); dots (circle), \(\gamma_f = 0.6\) and for the supporting bearing, solids (square), \(\gamma_f = 1\); dashes (square), \(\gamma_f = 0.8\); dots (square), \(\gamma_f = 0.6\).

Fig. 6. Locus of the global optimum of both the unbalance mass and the supporting bearing over different value of the capacity of a supporting bearing, \(\gamma_f\). For the unbalance mass, solids (circle), \(\gamma_m = 1\); dashes (circle), \(\gamma_m = 3\); dots (circle), \(\gamma_m = 5\) and for the supporting bearing, solids (square), \(\gamma_m = 1\); dashes (square), \(\gamma_m = 3\); dots (square), \(\gamma_m = 5\).

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Table 1
Specification of the target vehicle engine.

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crank radius ((r_C))</td>
<td>mm</td>
<td>49</td>
</tr>
<tr>
<td>Connecting rod pitch ((l_p))</td>
<td>mm</td>
<td>155</td>
</tr>
<tr>
<td>Piston mass ((\gamma))</td>
<td>g</td>
<td>757.4</td>
</tr>
<tr>
<td>Equivalent reciprocal mass except piston</td>
<td>g</td>
<td>426.9</td>
</tr>
</tbody>
</table>
optimum, the position of the support bearing, which is 0.2789 (calculated using Eq. (18)), is identical to that of the unbalance mass.

\[
\frac{x_{m, \gamma_f^{-1}}}{250} = \frac{\sqrt{\frac{3}{12(3+1)}}}{1/(1-(\sqrt{1+\frac{3}{3}}))}\sqrt{\frac{3}{12(3+1)}} = 0.2789, 
\]

(17)

\[
\frac{x_b, \gamma_f^{-1}}{250} = \frac{x_{m, \gamma_f^{-1}}}{250} = 0.2789. 
\]

(18)

Second, we simulate the local optimum of the objective function with a limited load capacity for the support bearing under the assumption that \( \gamma_f = 0.9 \). The locus of local minima for the objective function is shown in Fig. 8 and the best position for the unbalance mass, 0.2181, is identical to the global position derived in Eq. (14) with \( \gamma_f = 0.9 \), as shown in Eq. (19). However, the position of the unbalance mass at the global optimum (0.2181, see Eq. (19)) is less than that of the support bearing because the limited load capacity of the support bearing, as shown in Eq. (20).

\[
\frac{x_{m, \gamma_f^{-0.9}}}{250} = \frac{\sqrt{\frac{3}{12(3+1)}}}{1/(2 \times 0.9 - 1) - (\sqrt{1+\frac{3}{3}})\sqrt{\frac{3}{12(3+1)}}} = 0.2181, 
\]

(19)

\[
\frac{x_b, \gamma_f^{-0.9}}{250} = \frac{x_{m, \gamma_f^{-0.9}}}{250(2 \times 0.9 - 1)} = 0.2726. 
\]

(20)

Both simulation results reveal that the optimal position for unbalances can be predicted seamlessly using the explicit solution as defined in Eq. (14), even complementary for the unlimited load capacity of a supporting bearing. Correspondingly, the optimal position of the support bearing is equal or less than that of the unbalances as defined in Eq. (15), and its explicit position is given by the design parameter, \( \gamma_f \).
4. Conclusion

This paper presents an optimal conceptual design of a balance shaft by determining locations of both unbalance and supporting bearing. Optimal locations of both a support bearing and an unbalance mass were successfully derived with single optimal formulation that globally minimizes the normalized sum of the elastic strain energy and the kinematic energy of the balance shaft. The simulation result reveals that the determined locations of both the support bearing and the unbalance mass depended on the load capacity of the support bearing as well as the mass ratio between an asymmetric part and an asymmetric part. In particular, the optimal supporting bearing location is highly susceptible for the load capacity of the responsible bearing.

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