Ternary radial harmonic Fourier moments based robust stereo image zero-watermarking algorithm

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\textbf{A B S T R A C T}

With the development and popularization of computer network technology, the copyright protection of stereo images is a serious problem to be solved. Based on ternary number theory and radial harmonic Fourier moments (RHFMs), ternary radial harmonic Fourier moments (TRHFMs) is proposed to deal with stereo images in a holistic manner, and based on this moment, this paper proposes a robust stereo image zero-watermarking algorithm. We first compute the TRHFMs of the original stereo image, and we randomly select TRHFMs using logistic mapping; then, we obtain a binary feature image using the magnitudes of the selected TRHFMs, and finally, we apply a bitwise exclusive-or operation on permuted logo image and binary feature image to obtain the zero-watermark image. Experimental results indicate that the proposed stereo image zero-watermarking algorithm is strongly robust to various asymmetric and symmetric attacks and has superiority compared with other zero-watermarking algorithms.

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1. Introduction

Over the past few decades, stereo image processing technology has drawn attention to increasingly more research. Stereo images are based on binocular parallax of the human eye to obtain three-dimensional sense. The existence of binocular parallax makes the human brain perceive the depth of the information in an image and gives people a strong visual shock and a sense of being immersed [3]. However, stereo images are easy to copy and modify, which seriously infringes upon the copyright of the author. Therefore, the copyright protection for stereo images is a serious problem to be solved.

As is known, image watermarking technology is the main technology that is used to protect the copyright of planar images [28]. However, research on stereo image digital watermarking technology has only recently started. Based on the adaptive disparity matching algorithm and DCT, Lee et al. [13] proposed a novel stereo image watermarking algorithm for copyright protection, which improves the PSNR value of the extracted watermark image from the reconstructed image. Yu et al. [32] put forward a new stereo image watermarking algorithm that is based on block-relationships. This algorithm applies the intra-block and inter-block relationships for embedding watermarks. A semi-fragile digital watermarking algorithm based on the discrete wavelet transform (DWT) domain of the stereo image left and right views was presented by Campisi [4]. This algorithm is robust to JPEG2000 compression, but it is fragile to other malicious attacks. Based on fractional

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Fourier transform (FrFT) and singular value decomposition (SVD), Bhatnagar et al. [2] brought forward a robust stereo image watermarking algorithm, which has strong robustness and security. Lin et al. [17] designed a blind depth-image-based rendering (DIBR) 3D image watermarking algorithm, which has good robustness to some common image processing attacks. A more robust DIBR 3D image watermarking algorithm based on dual-tree complex wavelet transform (DT-CWT) was presented by Kim et al. [12], which achieves good imperceptibility. Wang et al. [27] proposed a watermarking algorithm for DIBR 3D images based on SIFT feature points, which is robust to some common image processing attacks. Cui et al. [8] presented a robust blind DIBR 3D images watermarking algorithm, which achieves good robustness to geometric attacks and common image processing attacks. Zhou et al. [33] proposed a novel binocular visual characteristics-based pixel-wise fragile watermarking algorithm, which is used for stereoscopic image authentication and locating tampered regions. Al-Haj et al. [1] came up with a 3D DIBR image watermarking algorithm based on the hybrid DWT and SVD transforms; the algorithm has good imperceptibility and robustness.

In addition, since the zero-watermarking algorithm [30] can provide good balance between the robustness and the imperceptibility, it has become one of the research hotspots in digital watermarking technology. Since zero-watermarking technology was proposed, many scholars have conducted in-depth research and proposed many excellent algorithms. The concept of zero-watermarking technology was proposed by Wen et al. [30]. These authors indicate that the zero-watermarking technology focuses on how to construct watermark information using image features rather than on how to embed watermark information into image features. Chen et al. [7] proposed a zero-watermarking system for public copyright authentication. That algorithm constructs a feature matrix from the low-frequency coefficients of the original image wavelet domain and uses it as a watermark matrix. This algorithm can effectively resist the interpretation attack and makes up for the shortcomings of the traditional zero-watermarking system. Chang and Lin [5] improved the feature extraction method on the basis of the algorithm in [7] and proposed a new zero-watermarking algorithm in the spatial domain. This scheme uses Sobel edge detection to extract the feature matrix. Gao and Jiang [10] designed and developed a robust visual zero-watermarking scheme based on Bessel-Fourier moments. The zero-watermark image is constructed using the magnitudes of the Bessel-Fourier moments. Chen et al. [6] proposed a zero-watermarking method for stereo images based on the texture features of the image block. First, the one-level DWT transform is performed on the left and right views of the stereo image, and the approximate coefficients of the DWT transform are calculated; then, the approximate coefficients are divided into non-overlap blocks and classified according to the texture of the block; finally, the block type relationship between the blocks is used to construct the zero-watermark information. Zhou et al. [34] proposed a zero-watermarking algorithm for stereo image copyright protection by studying the disparity stability of the DWT domain of the stereo image left and right views and the pseudo randomness of the hyperchaotic discrete system, which has good robustness to all types of symmetric and asymmetric attacks. Wang et al. [26] proposed a geometric attack-resistant color image zero-watermarking algorithm based on quaternion exponential moments (QEMs). Experimental results indicate that the proposed algorithm resists various attacks significantly better than similar zero-watermarking algorithms and the QEMs-based traditional watermarking algorithm. Later, they developed a new zero-watermarking algorithm [24] using logistic mapping and polar complex exponential transform (PCET). Logistic mapping is used to randomly select PCET coefficients in this algorithm, which can improve the security of the algorithm.

In summary, researchers have conducted some research on stereo image traditional watermarking technology and planar image zero-watermarking technology. However, there is little research on stereo image zero-watermarking algorithms. Moreover, most of them do not consider the intrinsic relationship between the left and right views of a stereo image, which lowers the performance of the zero-watermarking algorithm. In addition, they can only resist common image processing attacks and cannot resist geometric attacks effectively, such as rotation and scaling. To solve the above problems, based on ternary number theory and radial harmonic Fourier moments (TRHFM), ternary radial harmonic Fourier moments (TRHFM) is proposed here, and based on this moment, the present paper proposes a robust stereo image zero-watermarking algorithm. In this algorithm, TRHFM effectively deals with stereo images in a holistic manner, which preserves the special relationship between the left and right views. At the same time, TRHFM has good geometric invariance, which will effectively improve its capability in resisting geometric attacks. The TRHFM of the original stereo image is computed first, and the robust moments suitable for constructing a zero-watermark are selected. Then, a binary feature image is constructed by using the magnitudes of the selected robust moments. Finally, a bitwise exclusive-or operation between the binary feature image and the permuted binary logo image is performed to generate the zero-watermark image. Experiment results show that the proposed algorithm can effectively resist common image processing attacks and geometric attacks and show the superiority of the proposed algorithm compared with other zero-watermarking algorithms.

The remainder of this paper is organized as follows. Section 2 introduces the definition and properties of TRHFM. Section 3 presents the proposed stereo image zero-watermarking algorithm. Experimental analysis and results are discussed in Section 4. Finally, Section 5 concludes the paper.
2. Ternary radial harmonic Fourier moments of stereo images

2.1. Radial harmonic Fourier moments

The definition of the radial harmonic Fourier moments (RHFM) [22] is as follows:

\[ \phi_{nm} = \frac{1}{2\pi} \int_{0}^{2\pi} \int_{0}^{1} f(r, \theta) T_n(r) \exp(-jm\theta) r dr d\theta, \]  

(1)

where \( \phi_{nm} \) is the RHFM of order \( n(n \geq 0) \) with repetition \( m(|m| \geq 0) \), and \( T_n(r) \) is the radial basis function (RBF), which is defined as follows:

\[ T_n(r) = \begin{cases} \frac{1}{r} & \text{while } n = 0 \\ \frac{2}{r} \sin(n+1)\pi r, & \text{while } n \text{ is odd} \\ \frac{2}{r} \cos(n)\pi r, & \text{while } n \text{ is even} \end{cases} \]

(2)

The basic function \( P_{nm}(r, \theta) = T_n(r) \exp(jm\theta) \) is orthogonal in the unit circle:

\[ \int_{0}^{2\pi} \int_{0}^{1} P_{nm}(r, \theta) P_{n'm'}(r, \theta) r dr d\theta = 2\pi \delta_{nn'} \delta_{mm'}, \]  

(3)

where \( 0 \leq r \leq 1, 0 \leq \theta \leq 2\pi, P_{n'm'}(r, \theta) \) is the conjugate of \( P_{nm}(r, \theta) \), and \( 2\pi \) is the normalization factor.

The original image \( f(r, \theta) \) can be approximately reconstructed using limited RHFMs:

\[ f(r, \theta) = \sum_{n=0}^{n_{\text{max}}} \sum_{m=-m_{\text{max}}}^{m_{\text{max}}} \phi_{nm} T_n(r) \exp(jm\theta). \]

(4)

As proposed in [25], the RHFM can be computed using a fast and precise method in polar coordinates based on FFT. The specific procedure is as follows:

The rectangular coordinate image with \( N \times N \) pixels is converted to a polar coordinate image \( f_p(r_u, \theta_v) \) using the following formula:

\[ f_p(r_u, \theta_v) = f\left( -r_u \times \frac{N}{2} \times \sin\theta_v + \frac{N}{2} + 1, r_u \times \frac{N}{2} \times \cos\theta_v - \frac{N}{2} \right), \]

(5)

where \( r_u = \frac{u}{M}, \theta_v = \frac{2\pi v}{M}, u, v = 0, 1, ..., M - 1, M = 4N. \)

Then, the RHFM \( \phi_{nm} \) can be obtained using FFT, as follows:

\[ \phi_{0,m} = \sqrt{2} F\left( \frac{M}{2} + 1, \frac{M}{2} + 1 + m \right), \quad n = k = 0 \]

\[ \phi_{n=2k,m} = F\left( \frac{M}{2} + 1 + k, \frac{M}{2} + 1 + m \right) + F\left( \frac{M}{2} + 1 - k, \frac{M}{2} + 1 + m \right), \quad n = 2k, \ k = 1, 2, ... \]

\[ \phi_{n=2k-1,m} = j F\left( \frac{M}{2} + 1 + k, \frac{M}{2} + 1 + m \right) - F\left( \frac{M}{2} + 1 - k, \frac{M}{2} + 1 + m \right), \quad n = 2k - 1, \ k = 1, 2, ... \]

(6)

where \( F \) is the 2D Fourier domain (FFT) [9] of the function \( G_p(r_u, \theta_v) = f_p(r_u, \theta_v) \sqrt{2} \) by moving the zero-frequency component to the center of the array. In this paper, the fast and precise method of computing RHFM is used to construct the ternary radial harmonic Fourier moments.

2.2. Ternary radial harmonic Fourier moments

Similar to the quaternion [11,29], the ternary number is a generalization of a complex number, which has one real and two imaginary parts:

\[ p = a + bi + cj. \]

(7)

where \( a, b \) and \( c \) are real numbers, and \( i \) and \( j \) are complex operators. The magnitude of \( p \) is \( |p| = \sqrt{a^2 + b^2 + c^2}. \)

There exists a large amount of controversy on ternary number theory, and hence, there are not specific rules of the properties of the complex operators \( i \) and \( j \). In this paper, we assume that \( i \) and \( j \) satisfy the following properties:

\[ i^2 = j^2 = -1, \ ij = ji = 0. \]

(8)

By using the ternary number representation, a stereo image \( f(r, \theta) \) can be considered to be an array of pure ternary numbers:

\[ f(r, \theta) = f_L(r, \theta)i + f_R(r, \theta)j. \]

(9)
where \( f_L(r, \theta) \) and \( f_R(r, \theta) \) represent the left and right views of \( f(r, \theta) \), respectively.

The left-side ternary radial harmonic Fourier moments (TRHFM) of order \( n (n \geq 0) \) with repetition \( m (|m| \geq 0) \) of stereo image \( f(r, \theta) \) is as follows:

\[
\phi_{nm}^L = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \exp(-\mu m \theta) f(r, \theta) rdr \theta.
\]

(10)

where \( \mu \) is a pure unit ternary number chosen as \( \mu = (i + j)/\sqrt{2} \) in this paper.

According to orthogonal function theory, the original stereo image \( f(r, \theta) \) can be approximately reconstructed by the weighted superposition of the TRHFMs:

\[
f(r, \theta) = \sum_{n=0}^{n_{max}} \sum_{m=-m_{max}}^{m_{max}} T_n(r) \exp(\mu m \theta) \phi_{nm}^L.
\]

(11)

2.3. Relationship between the TRHFM and the complex RHFM for a single view of the stereo image

The relationship between the TRHFM computation and the complex RHFM computation for a single view of the stereo image is as follows:

\[
\phi_{nm}^L = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \exp(-\mu m \theta) f(r, \theta) rdr \theta
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \exp(-\mu m \theta) [f_L(r, \theta)i + f_R(r, \theta)]j rdr \theta
\]

\[
= \left[ \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \exp(-\mu m \theta) f_L(r, \theta) rdr \theta \right]i + \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \exp(-\mu m \theta) f_R(r, \theta) rdr \theta |j
\]

\[
= \left[ \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \cos(m \theta) f_L(r, \theta) rdr \theta - \mu \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \sin(m \theta) f_L(r, \theta) rdr \theta \right]i
\]

\[
+ \left[ \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \cos(m \theta) f_R(r, \theta) rdr \theta - \mu \frac{1}{2\pi} \int_0^{2\pi} \int_0^{1} T_n(r) \sin(m \theta) f_R(r, \theta) rdr \theta \right]j
\]

\[
= [\text{Re}(\phi_{nm}(f_L)) + \frac{(i + j)}{\sqrt{2}} \text{Im}(\phi_{nm}(f_L))]i + [\text{Re}(\phi_{nm}(f_R)) + \frac{(i + j)}{\sqrt{2}} \text{Im}(\phi_{nm}(f_R))]j
\]

\[
= A_{nm} + iB_{nm} + jC_{nm}
\]

(12)

with

\[
A_{nm} = -\frac{1}{\sqrt{2}} [\text{Im}(\phi_{nm}(f_L)) + \text{Im}(\phi_{nm}(f_R))]
\]

\[
B_{nm} = \text{Re}(\phi_{nm}(f_L))
\]

\[
C_{nm} = \text{Re}(\phi_{nm}(f_R)).
\]

(13)

where \( f_L \) and \( f_R \) represent the left and right views of the image \( f(r, \theta) \), respectively; \( \phi_{nm}(f_L) \) and \( \phi_{nm}(f_R) \) are the corresponding RHFM; \( \text{Re}(p) \) denotes the real part of the complex number \( p \); and \( \text{Im}(p) \) is the imaginary part.

Similar to the TRHFM computation, we have the following relationship between the TRHFM reconstruction and the complex RHFM reconstruction for a single view grayscale image:

\[
f'(r, \theta) = f'_A(r, \theta) + if'_B(r, \theta) + jf'_C(r, \theta)
\]

(14)

with

\[
f'_A(r, \theta) = \text{Re}(A'_{nm}) - \frac{1}{\sqrt{2}} [\text{Im}(B'_{nm}) + \beta \text{Im}(C'_{nm})]
\]

\[
f'_B(r, \theta) = \text{Re}(B'_{nm}) + \frac{1}{\sqrt{2}} \text{Im}(A'_{nm})
\]

\[
f'_C(r, \theta) = \text{Re}(C'_{nm}) + \frac{1}{\sqrt{2}} \text{Im}(A'_{nm}).
\]

(15)

where \( f'_A(r, \theta) \) is a matrix that is approximate to 0; \( f'_B(r, \theta) \) and \( f'_C(r, \theta) \) correspond to the left and right views of the reconstructed stereo image \( f'(r, \theta) \), respectively; and \( A'_{nm}, B'_{nm} \) and \( C'_{nm} \) are grayscale reconstruction matrices of \( A_{nm}, B_{nm} \) and \( C_{nm} \), respectively.
2.4. Performance of TRHFM reconstruction

It should be noted that since we assume $ij = ji = 0$ in this paper, some information will be lost in the TRHFM computation and reconstruction process, for example, the items $\frac{1}{\sqrt{2}} \text{Im}(\phi_{nm}(f_1))$ and $\frac{1}{\sqrt{2}} \text{Im}(\phi_{nm}(f_2))$ in the TRHFM computation process. This circumstance means that the reconstructed images using TRHFM will have extra errors.

Stereo image ‘Art’ from Middlebury 2005 Stereo Datasets [23] with 128 × 128 pixels is used to test the performance of TRHFM reconstruction. The left and right views of the stereo image ‘Art’ are shown Fig. 1. The reconstruction images for the left and right views of the stereo image ‘Art’ (max moment order $n_{\text{max}} = 5, 10, 20, 30, 40, 50$) are shown in Fig. 2.

As seen in Fig. 2, although there are some reconstruction errors, TRHFM can still reconstruct the original image well.

3. Zero-watermarking algorithm

3.1. Zero-watermark generation

By using ternary number theory, the stereo image can be treated as a vector field, and the zero-watermark image can be constructed directly using the TRHFM magnitudes. As is known, the exclusive-or operation satisfies the following rule:

$$A = \text{XOR}(\text{XOR}(A, B), B), A, B \in \{0, 1\}. \quad (16)$$

Hence, the exclusive-or operation is used for the zero-watermark generation procedure and zero-watermark verification procedure, which can achieve the blind watermark verification.

Let $I = \{f(x, y), 1 \leq x, y \leq N\}$ be the original grayscale stereo image, and let $L = \{l(i, j), 1 \leq i, j \leq Q\}$ be the binary logo image. The flow chart of the zero-watermark generation procedure is shown in Fig. 3 and is concluded to be as follows.

**Step 1: Logo image permuting**

To improve the robustness of the zero-watermarking algorithm, the affine transform [19] is used for a two-dimensional pseudorandom permutation of the logo image $L$ with seed s1, and we obtain the permuted logo image $L_p = \{l_p(i, j), 1 \leq i, j \leq Q\}$. The definition of the affine transform is as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}, 0 \leq x, y < Q. \quad (17)$$

where $(x, y)$ and $(x', y')$ are the original logo image pixel and the permuted logo image pixel, respectively; $a, b, c, d$ denote the deformation coefficients; and $e, f$ represent the translation coefficients.
Step 2: TRHFM computation of the original grayscale stereo image

The \( n_{\text{max}} \)-order TRHFM of the original grayscale stereo image \( I \) is computed using Eq. (10), which can obtain \((n_{\text{max}} + 1)(2n_{\text{max}} + 1)\) TRHFMs.

Step 3: Feature vector construction

To improve the safety performance of the algorithm, logistic mapping [20] with secret key \( K \) is used to select \( Q^2 \) TRHFMs \( \phi^i = \{\phi^i_1, \phi^i_2, \ldots, \phi^i_{Q^2}\} \) from the \((n_{\text{max}} + 1)(2n_{\text{max}} + 1)\) TRHFMs to construct the zero-watermark image. The detailed steps are described below:

Logistic mapping has the following form:

\[
x_{i+1} = \lambda x_i (1 - x_i) (0 < x_i \leq 1).
\]

where \( \lambda \) is the control parameter, and \( x_i \) is the chaos sequence. Let \( X_i = \text{int}(Z \times x_i) \), \( 1 \leq i \leq Q^2 \) be the indices of the selected TRHFMs, where \( Z = (n_{\text{max}} + 1)(2n_{\text{max}} + 1) \). Then, the coordinates \( X_i \) of the \( Q^2 \) selected TRHFMs can be obtained. Finally, the magnitudes of the selected TRHFMs are used to construct the feature vector \( \vec{M} = \{M_1, M_2, \ldots, M_{Q^2}\} \).

Step 4: Obtaining the feature image

Let threshold \( T \) be the mean of the feature vector \( \vec{M} \); then, the binary feature vector \( \vec{B} = \{B_1, B_2, \ldots, B_{Q^2}\} \) can be obtained using the following binarization processing:

\[
B_i = \begin{cases} 
1, & \text{if } M_i \geq T \\
0, & \text{if } M_i < T 
\end{cases}
\]

Then, the feature vector \( \vec{B} \) is converted to a 2D feature image \( LF \) with \( Q \times Q \) pixels, and the feature image \( LF \) is permuted using the affine transform with seed \( s_2 \); then, we obtain the permuted feature image \( LF_p \).

Step 5: Zero-watermark image generation

The exclusive-or operation is used to generate the zero-watermark image \( W_v \), as follows:

\[
W_v = \text{XOR}(LF_p, L_p).
\]

where \( LF_p \) is the permuted feature image, and \( L_p \) is the permuted logo image.

Finally, the zero-watermark image \( W_v \), the seed \( s_1 \) in Step 1, the secret key \( K \) in Step 3 and the seed \( s_2 \) in Step 4 are stored in the intellectual property databases for copyright protection.

3.2. Zero-watermark verification

The zero-watermark verification procedure is used to validate the copyright of the protected grayscale stereo image \( I^* \), and the detailed procedure is summarized as follows:

Step 1: TRHFM computation of the protected grayscale stereo image

The \( n_{\text{max}} \)-order TRHFM of the protected grayscale stereo image \( I^* \) is computed using Eq. (10), which can obtain \((n_{\text{max}} + 1)(2n_{\text{max}} + 1)\) TRHFMs.

Step 2: Feature vector construction

\( Q^2 \) TRHFMs from the \((n_{\text{max}} + 1)(2n_{\text{max}} + 1)\) TRHFMs are selected using the logistic mapping with secret key \( K \), and then, the magnitudes of the selected TRHFMs are used to construct the feature vector \( \vec{M}' = \{M'_1, M'_2, \ldots, M'_{Q^2}\} \).

Step 3: Feature image obtaining

The feature vector \( \vec{M}' \) is converted to the binary feature vector \( \vec{B}' = \{B'_1, B'_2, \ldots, B'_{Q^2}\} \) using binarization processing. Then, the feature vector \( \vec{B}' \) is converted to a 2D feature image \( LF^* = \{f^*(i, j) \}, 1 \leq i, j \leq Q \), and the feature image \( LF^* \) is then permuted using the affine transform with seed \( s_2 \), and the permuted feature image \( LF_p^* = \{f_p^*(i, j) \}, 1 \leq i, j \leq Q \) is obtained.

Step 4: Verified logo image generation
Fig. 4. The left and right views of ‘Art’, ‘Books’, ‘Computer’ and ‘Dolls’ and the binary logo image. (a)-(d) Left views, (e)-(h) Right views, (i) The binary logo image.

The permuted logo image \( L_p^* = \{l_p^*(i, j), 1 \leq i, j \leq Q\} \) is generated from the permuted feature image \( LF_p^* \) and the zero-watermark image \( W_v \) based on the following exclusive-or operation:

\[
L_p^* = \text{XOR}(LF_p^*, W_v).
\]  

Finally, the verified logo image \( L^* = \{l^*(i, j), 1 \leq i, j \leq Q\} \) is obtained after an inverse affine transform on \( L_p^* \) using seed s1.

4. Experimental results

Twenty grayscale stereo images from Middlebury 2005 Stereo Datasets [23] with 512 × 512 pixels and one binary logo image with 64 × 64 pixels are used to investigate the performance of the proposed algorithm. The left and right views of the four grayscale stereo images ‘Art’, ‘Books’, ‘Computer’ and ‘Dolls’ and the binary logo image are shown in Fig. 4. The max moment order of TRHFM is set to \( n_{\text{max}} = 50 \).

4.1. Robustness to various attacks

The proposed algorithm is suitable for stereo images that have two views, and hence, there are two types of image attacks: asymmetric attacks and symmetric attacks [34]. Asymmetric attacks denote that only one view of the stereo image is attacked, while symmetric attacks explain that the two views of the stereo image are attacked simultaneously. Unless stated, the left view of the stereo image is used to test for an asymmetric attack in this paper. The quality of the attacked image is evaluated by the peak signal-to-noise ratio (PSNR) [14,29]. PSNR_L and PSNR_R are used to denote the PSNR of the left and right views of the attacked image, respectively. The definition of PSNR is as follows:

\[
\text{PSNR} = 10 \log_{10} \frac{255^2}{\text{MSE}}.
\]  

(22)
Table 1
Robustness to image rotation.

<table>
<thead>
<tr>
<th></th>
<th>Art</th>
<th>Books</th>
<th>Computer</th>
<th>Dolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation with cropping 5°</td>
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<td></td>
<td>PSNRR(db)</td>
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<td>Asymmetrical attack BER</td>
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<td>0.0449</td>
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<td>Symmetrical attack BER</td>
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<td>Rotation with cropping 45°</td>
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<tr>
<td>Asymmetrical attack BER</td>
<td>0.0481</td>
<td>0.0415</td>
<td>0.0449</td>
<td>0.0425</td>
</tr>
<tr>
<td>Symmetrical attack BER</td>
<td>0.0562</td>
<td>0.0525</td>
<td>0.0588</td>
<td>0.0452</td>
</tr>
</tbody>
</table>

where MSE is the mean square error

$$\text{MSE} = \frac{1}{N^2} \sum_{x=1}^{N} \sum_{y=1}^{N} [f(x, y) - f^*(x, y)]^2,$$

where $f(x, y)$ and $f^*(x, y)$ denote the original image and the attacked image, respectively. Their size is $N \times N$.

The robustness of the proposed algorithm is evaluated by the bit error rate (BER) \cite{31} between the verified logo image and the original logo image. The BER is defined as

$$\text{BER} = \frac{\text{Number of erroneously detected bits}}{\text{Watermark length}} \times 100\%.$$

4.1.1. Robustness to image rotation

Image rotation is a common geometric attack that changes the position of the image pixels. The original stereo image was rotated with cropping by 5° and 45°, and the results for asymmetric and symmetric attacks are shown in Table 1. We can observe from this table that the BER values are lower than 0.07, which proves that the proposed algorithm can effectively resist image rotation.

4.1.2. Robustness to image scaling

Next, the robustness against image scaling is tested. We scaled the original stereo image with factors of 0.25 and 4, and then, we rescaled them back to the original size. Table IV presents the BER values under image scaling. The results show that the BER values are less than 0.02, which indicates that the proposed algorithm has strong robustness against image scaling (Table 2).

4.1.3. Robustness to image cropping

We also test the performance of the proposed algorithm under image cropping. We applied upper-left-corner cropping (1/16 and 1/8) on the original stereo image. Table 3 presents the BER value for Art, Books, Computer and Dolls. The results show that the image cropping has no effect on the performance of the proposed algorithm.

4.1.4. Robustness to JPEG compression

In Table 4, we test the robustness of the proposed algorithm when the original stereo images undergo JPEG compression with a quality factor of $Q = 10, 30, 50, 70, 90$. As seen from this table, the proposed algorithm still has very low BER values when $Q = 10$, which are less than 0.03. That means that the proposed algorithm provides good performance under JPEG compression attacks.

4.1.5. Robustness to median and Gaussian filtering

Another experiment is conducted to test the robustness of the proposed algorithm under median and Gaussian filtering with window sizes of $3 \times 3$. The results for the original stereo images of Art, Books, Computer and Dolls are given in Table 5, which indicate that the proposed algorithm has strong capability against median and Gaussian filtering.
4.1.6. Robustness to Gaussian and salt-and-pepper noise

Finally, the robustness of the proposed algorithm is tested in terms of Gaussian noise with variance 0.01 and salt-and-pepper noise with density of 0.03. Table 6 shows that the proposed algorithm achieves superb robustness against Gaussian and salt-and-pepper noise.

4.2. Comparison with similar zero-watermarking algorithms

The performance of the proposed stereo image zero-watermarking algorithm is compared with three superb zero-watermarking algorithms [6,24,34], among which [6,34] are two stereo image zero-watermarking algorithms, and the algorithm in [24] is a superb planar image zero-watermarking algorithm. The average value of the BERs of twenty grayscale stereo images in Fig. 4 is used to compare the performance of the four algorithms. In the experiments, symmetrical attacks are used to evaluate the robustness of the stereo image zero-watermarking algorithms. For the planar image zero-watermarking algorithm, the left view of the stereo image is used as the original image to evaluate the robustness.

The attack methods include geometric attacks and common image processing attacks, which are Rotation with cropping 5°, Rotation with cropping 45°, Scaling 0.25 and rescaling to the original size, Scaling 4 and rescaling to the original size, Upper left corner cropping (1/16), Upper left corner cropping (1/8), JPEG compression (10), JPEG compression (30), JPEG compression (50), Median filtering (3 × 3), Median filtering (3 × 3), Gaussian filtering (3 × 3), Gaussian filtering (3 × 3), and the experiments are performed on fifty grayscale stereo images.

Table 3
Robustness to image cropping.

<table>
<thead>
<tr>
<th>Method</th>
<th>Art</th>
<th>Books</th>
<th>Computer</th>
<th>Dolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper left corner cropping (1/16)</td>
<td>PSNRL(db)</td>
<td>25.8210</td>
<td>27.2419</td>
<td>26.2083</td>
</tr>
<tr>
<td></td>
<td>PSNR(db)</td>
<td>25.3419</td>
<td>27.0781</td>
<td>26.4739</td>
</tr>
<tr>
<td></td>
<td>Asymmetrical attack BER</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Symmetrical attack BER</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4
Robustness to JPEG compression.

<table>
<thead>
<tr>
<th>Method</th>
<th>Art</th>
<th>Books</th>
<th>Computer</th>
<th>Dolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPEG compression (10)</td>
<td>PSNRL(db)</td>
<td>30.3952</td>
<td>28.9560</td>
<td>29.9570</td>
</tr>
<tr>
<td></td>
<td>PSNR(db)</td>
<td>30.6761</td>
<td>28.6581</td>
<td>29.6921</td>
</tr>
<tr>
<td></td>
<td>Asymmetrical attack BER</td>
<td>0.0093</td>
<td>0.0073</td>
<td>0.0171</td>
</tr>
<tr>
<td></td>
<td>Symmetrical attack BER</td>
<td>0.0166</td>
<td>0.0173</td>
<td>0.0242</td>
</tr>
</tbody>
</table>

Table 5
Robustness to median and Gaussian filtering.

<table>
<thead>
<tr>
<th>Method</th>
<th>Art</th>
<th>Books</th>
<th>Computer</th>
<th>Dolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median filtering (3 × 3)</td>
<td>PSNRL(db)</td>
<td>37.0048</td>
<td>31.7205</td>
<td>33.5259</td>
</tr>
<tr>
<td></td>
<td>PSNR(db)</td>
<td>37.7685</td>
<td>31.0040</td>
<td>33.0953</td>
</tr>
<tr>
<td></td>
<td>Asymmetrical attack BER</td>
<td>0.0029</td>
<td>0.0054</td>
<td>0.0042</td>
</tr>
<tr>
<td></td>
<td>Symmetrical attack BER</td>
<td>0.0049</td>
<td>0.0068</td>
<td>0.0085</td>
</tr>
<tr>
<td>Gaussian filtering (3 × 3)</td>
<td>PSNRL(db)</td>
<td>42.4695</td>
<td>38.5031</td>
<td>40.4632</td>
</tr>
<tr>
<td></td>
<td>PSNR(db)</td>
<td>42.9432</td>
<td>38.0006</td>
<td>39.8700</td>
</tr>
<tr>
<td></td>
<td>Asymmetrical attack BER</td>
<td>0.0002</td>
<td>0.0005</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td>Symmetrical attack BER</td>
<td>0.0009</td>
<td>0.0015</td>
<td>0.0034</td>
</tr>
</tbody>
</table>
Table 6
Robustness to Gaussian and salt-and-pepper noise.

<table>
<thead>
<tr>
<th>Attack</th>
<th>Art</th>
<th>Books</th>
<th>Computer</th>
<th>Dolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian noise (0.01)</td>
<td>PSNR(dB)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSNR(dB)</td>
<td>20.5016</td>
<td>20.1811</td>
<td>20.1170</td>
<td>20.4390</td>
</tr>
<tr>
<td>Asymmetrical attack BER</td>
<td>0.0154</td>
<td>0.0144</td>
<td>0.0168</td>
<td>0.0107</td>
</tr>
<tr>
<td>Symmetrical attack BER</td>
<td>0.0198</td>
<td>0.0208</td>
<td>0.0276</td>
<td>0.0193</td>
</tr>
<tr>
<td>Salt-and-pepper noise (0.03)</td>
<td>PSNR(dB)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSNR(dB)</td>
<td>19.9065</td>
<td>20.5527</td>
<td>20.7909</td>
<td>20.4644</td>
</tr>
<tr>
<td>Asymmetrical attack BER</td>
<td>0.0149</td>
<td>0.0149</td>
<td>0.0178</td>
<td>0.0112</td>
</tr>
<tr>
<td>Symmetrical attack BER</td>
<td>0.0273</td>
<td>0.0220</td>
<td>0.0330</td>
<td>0.0208</td>
</tr>
</tbody>
</table>

Table 7
Robustness comparison between the proposed algorithm and other zero-watermarking algorithms.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation with cropping 5°</td>
<td>0.0554</td>
<td>0.5134</td>
<td>0.9016</td>
<td>0.0059</td>
</tr>
<tr>
<td>Rotation with cropping 45°</td>
<td>0.0513</td>
<td>0.4926</td>
<td>0.4956</td>
<td>0.0052</td>
</tr>
<tr>
<td>Scaling 0.25 and rescaling to the original size</td>
<td>0.0159</td>
<td>0.0535</td>
<td>0.0301</td>
<td>0.0527</td>
</tr>
<tr>
<td>Scaling 4 and rescaling to the original size</td>
<td>0.0005</td>
<td>0.0433</td>
<td>0.0315</td>
<td>0.0040</td>
</tr>
<tr>
<td>Upper left corner cropping (1/16)</td>
<td>0.0000</td>
<td>0.0467</td>
<td>0.0415</td>
<td>0.0000</td>
</tr>
<tr>
<td>Upper left corner cropping (1/8)</td>
<td>0.0000</td>
<td>0.0600</td>
<td>0.0378</td>
<td>0.0000</td>
</tr>
<tr>
<td>JPEG compression (10)</td>
<td>0.0160</td>
<td>0.0833</td>
<td>0.0952</td>
<td>0.0332</td>
</tr>
<tr>
<td>JPEG compression (30)</td>
<td>0.0042</td>
<td>0.0567</td>
<td>0.0742</td>
<td>0.0059</td>
</tr>
<tr>
<td>JPEG compression (50)</td>
<td>0.0028</td>
<td>0.0300</td>
<td>0.0688</td>
<td>0.0062</td>
</tr>
<tr>
<td>JPEG compression (70)</td>
<td>0.0015</td>
<td>0.0176</td>
<td>0.0573</td>
<td>0.0026</td>
</tr>
<tr>
<td>JPEG compression (90)</td>
<td>0.0000</td>
<td>0.0100</td>
<td>0.0535</td>
<td>0.0013</td>
</tr>
<tr>
<td>Median filtering (3 × 3)</td>
<td>0.0051</td>
<td>0.0667</td>
<td>0.0332</td>
<td>0.0147</td>
</tr>
<tr>
<td>Gaussian filtering (3 × 3)</td>
<td>0.0010</td>
<td>0.0200</td>
<td>0.0176</td>
<td>0.0052</td>
</tr>
<tr>
<td>Gaussian noise (0.01)</td>
<td>0.0200</td>
<td>0.0535</td>
<td>0.0793</td>
<td>0.0430</td>
</tr>
<tr>
<td>Salt-and-pepper noise (0.03)</td>
<td>0.0234</td>
<td>0.0467</td>
<td>0.0818</td>
<td>0.0400</td>
</tr>
</tbody>
</table>

compression (50), JPEG compression (70), JPEG compression (90), Median filtering (3 × 3), Gaussian filtering (3 × 3), Gaussian noise (0.01) and Salt-and-pepper noise (0.03). The comparison results are concluded in Table 7. It can be seen clearly that the proposed algorithm proves better robustness than the algorithms in [6,24,34] on the whole and is only slightly worse than [24] on resisting rotation with cropping attacks.

5. Conclusions

This paper proposes ternary radial harmonic Fourier moments (TRHFM) for stereo images based on ternary number theory and radial harmonic Fourier moments (RHFHM). Based on TRHFM, we propose a robust stereo image zero-watermarking algorithm, which implements copyright protection for stereo images. Experimental results show that the proposed algorithm is strongly robust to various asymmetric and symmetric attacks and has better performance compared with other zero-watermarking algorithms. The superiority of the proposed algorithm includes the following: (1) ternary number theory is used in stereo image processing for the first time; (2) TRHFM is applied to a stereo image zero-watermarking algorithm for the first time; (3) TRHFM is used for the image features to construct the zero-watermark image, which improves the robustness against asymmetric and symmetric attacks; and (4) the proposed algorithm is based on logistic mapping and is sensitive to the initial value of the chaotic system, which leads to excellent security. Accordingly, the proposed algorithm has important theoretical significance and practical value in the copyright protection of stereo images.

In future work, TRHFM can be applied in stereo image retrieval [16], stereo image segmentation [21], stereo image feature selection [15], stereo image reversible data hiding [18] and other research fields.

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