A Survey on Energy-Efficient Train Operation for Urban Rail Transit

Xin Yang, Xiang Li, Bin Ning, Fellow, IEEE, and Tao Tang

Abstract—Due to rising energy prices and environmental concerns, the energy efficiency of urban rail transit has attracted much attention from both researchers and practitioners in recent years. Timetable optimization and energy-efficient driving, as two mainly used train operation methods in relation to the tractive energy saving, make major contributions in reducing the energy consumption that has been studied for a long time. Generally speaking, timetable optimization synchronizes the accelerating and braking actions of trains to maximize the utilization of regenerative energy, and energy-efficient driving optimizes the speed profile at each section to minimize the tractive energy consumption. In this paper, we present a fully comprehensive survey on energy-efficient train operation for urban rail transit. First, a general energy consumption distribution of urban rail trains is described. Second, the current literature on timetable optimization and energy-efficient driving is reviewed. Finally, according to the review work, it is concluded that the integrated optimization method jointly optimizing the timetable and speed profile has become a new tendency and ought to be paid more attention in future research.

Index Terms—Train operation, urban rail transit, timetable optimization, energy-efficient driving, integrated optimization.

I. INTRODUCTION

A. Motivation

Due to its high capacity, safety and reliability, the urban rail transit has received rapid development around the world [1]. For example, in China, only 3 cities have 5 urban rail lines in 2000, while it has increased to 22 cities with 88 urban rail lines and a total distance over 3000 kilometers by the end of 2014 [2]. With the rising energy prices and environmental concerns, the energy efficiency problem has attracted much attention from both researchers and practitioners in recent years.

The gross energy consumption in the urban rail transit systems is considerable. Around 50% of the energy is consumed by trains, and the rest is used by infrastructure facilities (e.g., stations, depots, groundwater pumps, tunnel ventilation fans, tunnel lighting, etc.) to ensure the proper operation [3], [4]. Take the Beijing Metro Line 10 as an example. About 85.23 million kWh of energy is consumed in 2012, and 46.01 million kWh of it is used by the train [5]. The energy consumption of other main urban rail lines in Beijing is provided in Fig. 1, which shows the energy consumption is considerable so that the energy efficiency studies are meaningful.

B. Technical Term

For a good understanding of this paper, some technical terms are listed below.

- System's gross energy consumption: The energy consumed by the whole urban rail line including trains and infrastructure facilities (e.g., stations, depots, groundwater pumps, tunnel ventilation fans, tunnel lighting, etc.).
- Train's total energy consumption: The energy consumed by the train including the traction system for accelerating the train and by the auxiliary system for lighting, heating, air-conditioning, door operation, etc.
- Tractive energy consumption: The energy consumed by the traction system on the train for accelerating the train.
- Net energy consumption: The tractive energy consumption of the train once any energy recovered via regenerative braking has been taken in to account.
C. Contribution

Timetable optimization and energy-efficient driving, as two mainly used energy-efficient train operation methods in relation to the tractive energy, make large contributions on reducing the energy consumption and have been studied for a long time. Generally speaking, timetable optimization synchronizes the actions of trains to maximize the utilization of regenerative energy based on the accelerating and braking time provided by speed profiles, and energy-efficient driving optimizes the speed profiles at sections to minimize the tractive energy consumption under the timetable constraints. As shown in Fig. 2, based on the power network data, line data, operation data and vehicle data, we can obtain the optimized timetable and speed profile.

However, timetable optimization and energy-efficient driving are not completely independent. They are even closely related. The former provides the running time at each section to the latter, and the latter optimizes the accelerating and braking time at each section to the former.

With the intention of providing an appropriate research direction, the main contribution of this paper is to present a fully comprehensive survey on energy-efficient train operation. The remainder of the paper is organized as follows. A general description of energy consumption in urban rail trains is given in Section II. The current literature on timetable optimization method is reviewed in Section III, and the current literature on energy-efficient driving method is reviewed in Section IV. In Section V, according to the review work, it is concluded that the integrated optimization method jointly optimizing the timetable and speed profile will be a tendency in future research.

II. ENERGY CONSUMPTION IN URBAN RAIL TRAINS

In order to make a good understanding on energy-efficient train operation, this section describes how energy is consumed in trains.

First, we introduce the concept of regenerative braking that has been well applied in urban rail trains. Regenerative braking (Hasegawa and Uchida [6]) is an energy recovery mechanism that converts the kinetic energy during braking into electricity. This is in contrast to the conventionally mechanical braking, where the extra kinetic energy is converted into heat by friction in the brake linings and therefore wasted. In urban rail trains, the most common form of regenerative braking involves using an electric motor as an electric generator during the braking phase of trains to recover the kinetic energy into electricity. The principle of regenerative braking is shown in Fig. 3.

The energy consumption of urban rail trains is divided into two parts [7], [8] as shown in upper half of Fig. 4. An important part (around 20%) is used by auxiliary system of trains. The auxiliary system mainly includes ventilation, air-conditioning and illumination equipments, etc. The proportion of this part is influenced by the weather and climate conditions, which has little to do with train operation strategies. The rest part (around 80%) is mainly consumed by traction system of trains, the amount of which depends on train operation strategies. Take an urban rail train (BR 425) [9] as an example. During this part consumed by the traction system, as shown in lower half of Fig. 4, the regenerative energy accounts for 33.6%, the losses during acceleration phase are 24%, the energy consumed to overcome the tractive resistance of the vehicle during all phases is 13.6%, and the losses during braking phase are 8.8%.

III. TIMETABLE OPTIMIZATION

In this section, we introduce the timetable optimization method. Firstly, the timetable optimization problem is described in detail. Then a comprehensive review of the related literature is given. According to literature [10], the regenerative energy is primarily used to supply power for auxiliary systems of the braking train itself, such as the ventilation, air-conditioning and illumination equipments. If the train is equipped with on-board storage devices such as super-capacitors, some energy will be
absorbed by them. Note that the energy storage devices are generally of limited capacity and high cost, most trains operated in real-world systems are not equipped with them. The surplus energy accounting for most of regenerative energy is fed back into the overhead contact line, which can be immediately used by accelerating other trains located in the same electricity supply interval. The process of regenerative energy flowing from braking train to accelerating train is shown in Fig. 5. However, if any feedback energy cannot be utilized timely, for example, there is no accelerating trains at that time, it will be wasted by heating resistors or absorbed by wayside storage devices (if any) installed on the overhead contact line. The energy absorbed by storage devices can be reused later. To summarize, the distribution of regenerative energy is described in Fig. 6.

Literature [10] concluded that maximizing the regenerative energy exchange between trains is a preferential measure to utilize the regenerative energy in urban rail transit. Timetable optimization aims at finding the optimal timetable such that the regenerative energy can be furthest used to accelerate other trains.

A. Problem Description

In urban rail transit, coordinating accelerating and braking actions of trains by means of timetable optimization is a straightforward way to improve the utilization of regenerative energy [10]. As shown in Fig. 7:

- The trapezoidal curves above the horizontal axis denote trains’ power profile at acceleration phase, during which trains need to absorb energy from the overhead contact line. The energy consumption is positive.
- There is no energy consumed or regenerated during coasting phase.
- The trapezoidal curves below the horizontal axis denote trains’ power profile at braking phase, during which trains regenerate energy and feed back it into the overhead contact line that can be used to accelerate other trains. The energy consumption is negative.
- The overlapping area denotes the amount of utilized regenerative energy.

Compared to profile 1 of train $i$, profile 2 of which is generated by moving the profile 1 to left achieves more overlap with the profile of train $j$. It is seen that the overlapping area (denoting the amount of utilized regenerative energy) is different when the relative positions of two trains’ power-time profiles have a change. In fact, the position of power profiles of train $i$ and $j$ are decided by their departures and arrivals. Timetable optimization
aims to find the optimal departures and arrivals of trains to maximize the utilization of regenerative energy.

As stated in literature [11], [12], we define \( a = \{ a_i, 1 \leq i \leq I, 1 \leq n \leq N \} \) as the set of arrival time, where \( a_i \) denotes the time that train \( i \) arrives at station \( n \), \( I \) denotes the number of trains, and \( N \) denotes the number of stations; and define \( d = \{ d_{n}, 1 \leq i \leq I, 1 \leq n \leq N \} \) as the set of departure time, where \( d_{n} \) denotes the time that train \( i \) departs from station \( n \).

The total utilization of regenerative energy can be formulated as

\[
F(a, d) = \sum_{i=1}^{T} \sum_{s=1}^{N_s} \min \left\{ \sum_{i=1}^{I} w_i(a, d, t) \lambda(i, t, s), \sum_{i=1}^{I} f_i(a, d, t) \lambda(i, t, s) \right\}
\]

(1)

where \( T \) denotes the total operation time, \( N_s \) denotes the number of electricity supply intervals, \( w_i(a, d, t) \) denotes the energy regenerated from braking train \( i \) at time unit \([t, t+1] \), \( f_i(a, d, t) \) denotes the required energy to accelerate train \( i \) at time unit \([t, t+1] \), and \( \lambda(i, t, s) \) denotes whether train \( i \) is located in the electricity supply interval \( s \) at time \( t \) or not, i.e.,

\[
\lambda(i, t, s) = \begin{cases} 
1, & \text{if train } i \text{ is located in electricity} \\
0, & \text{supply interval } s \text{ at time } t \\
\end{cases}
\]

(2)

Generally speaking, the objective of most timetable optimization models is to maximize the total utilization of regenerative energy \( F(a, d) \).

### B. Literature Overview

Over the past decades, some timetable optimization methods on improving the regenerative energy utilization or reducing energy consumption have been studied.

1) **Limiting the Peak Power Consumption:** In first stage, researchers were interested in reducing the peak power consumption by limiting the peak voltage. In real-world systems, the overhead contact line is set to a threshold voltage for protecting the safety of the power network. During the trains’ regenerative braking process, the voltage of the overhead contact line increases continually. If the voltage reaches the threshold value, the regenerative braking will be removed and replaced by the mechanical braking. With mechanical braking mode, the kinetic energy is converted into heat by friction in the brake linings and it is wasted. Therefore, limiting the peak of power consumption can increase the percentage of regenerative braking in the whole braking process, which can achieve the goal of energy saving. For example, Gordon and Lehrer [13] firstly considered the regenerative braking energy by coordinating control of multiple trains in the Bay Area Rapid Transit system. They designed a control algorithm to reduce peak power consumption, avoid oscillations and limit needle peaks in power demand at substations by coordinating trains’ movements. Albrecht [14] presented a new approach based on dynamic programming to control running time of trains with a given optimal combination of headway and synchronization time, which was contributed to reducing power peaks and energy consumption. Chen et al. [15] employed the genetic algorithm to optimize train scheduling for avoiding the simultaneous acceleration of too many trains, such that the peak power consumption can be reduced.

2) **Maximizing the Utilization of Regenerative Energy:** In the second stage, researchers began to formulate the optimization model to maximize the utilization of regenerative energy directly. For example, Ramos et al. [16] presented a particular case of train timetabling problem during off-peak hours, which aimed at maximizing the overlapping time between speed-up and slow-down actions of all the trains located in the same electricity supply interval, so that the regenerative energy could be utilized more efficiently. They firstly took the overlapping time to measure the utilization of regenerative energy. Nasri et al. [17] proposed an optimized timetable of trains’ movement to completely utilize the regenerative energy from braking trains. The effect of headway and reserve times on the amount of energy consumption is studied and an optimized reserve time set is found for the sample system. Kim et al. [18] proposed a multi-criteria mixed integer programming to minimize the peak energy and simultaneously to maximize the utilization of regenerative energy. They coordinated the train departure times at the starting station and maintained the planned traveling time between stations. Peña-Alcaraz et al. [19] dealt with the design of underground rail system timetables that synchronize the movement of trains to allow the energy consumption from substations to be reduced by maximizing the use of regenerative braking energy. They developed a power flow model to calculate the regeneration saving factor of each synchronization within the electrical network. The most fascinating aspect of this paper is that they tested the proposed model in the Madrid Metro Line 3, and the results showed that 3.52% energy saving was obtained. Fournier et al. [20] developed an optimization model to maximize the utilization of regenerative energy by subtly modifying dwell time for trains at stations. A hybrid genetic/linear programming algorithm was implemented to solve this problem.

In 2013, Yang et al. [11] proposed a cooperative scheduling model to coordinate the accelerating and braking processes of adjacent trains, such that the regenerative energy from the braking trains can be immediately used by the accelerating trains. A simulation study based on the real-life data from the Beijing Metro Yizhuang Line showed that the cooperative scheduling model can significantly improve the overlapping time around 22%. Furthermore, Li and Yang [12] developed a stochastic cooperative scheduling model taking the randomness of departure delay for trains at busy stations into account, which could save energy around 8% compared with the cooperative scheduling approach [11]. In 2014, Yang et al. [21] proposed a timetable optimization model to coordinate up trains and down trains at the same station for improving the utilization of recovery energy and reducing the passenger waiting time. They conducted numerical examples based on the operation data from the Beijing Metro Yizhuang Line, and the results illustrated that the proposed model can save energy by 8.86% and reduce passenger waiting time by 3.22% in comparison with the current timetable. Inspired by the approach in literature [11], Zhao et al. [22] proposed a two-objective optimization model...
TABLE I
RECENT PUBLICATIONS ON TIMETABLE OPTIMIZATION

<table>
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<tr>
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<th>Model</th>
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<td>Peak power consumption</td>
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<td>Albrecht [14]</td>
<td>Peak power consumption</td>
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<tr>
<td>Chen et al. [15]</td>
<td>Peak power consumption</td>
<td>General programming model</td>
<td>Genetic algorithm</td>
<td>Energy saving of 28.8% in Kaohsiung Rapid Transit</td>
</tr>
<tr>
<td>Ramos et al. [16]</td>
<td>Overlapping time</td>
<td>Mixed integer programming model</td>
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<td>Overlap increase of 1.36 hours in Madrid Metro</td>
</tr>
<tr>
<td>Nasri et al. [17]</td>
<td>Utilization of regenerative energy</td>
<td>Integer programming model</td>
<td>Genetic algorithm</td>
<td>Energy saving of 10-14% in simulations at laboratory</td>
</tr>
<tr>
<td>Kim et al. [18]</td>
<td>1Utilization of regenerative energy 2Peak energy</td>
<td>Mixed integer programming model</td>
<td>Heuristics algorithm</td>
<td>Objective 1 improves 5% and objective 2 reduces 40% in Korea Metropolitan Subway</td>
</tr>
<tr>
<td>Peña-Alcaraz et al. [19]</td>
<td>Utilization of regenerative energy</td>
<td>Mixed integer programming model</td>
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<td>Energy saving of 3-7% in Madrid Metro Line 3</td>
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<tr>
<td>Fournier et al. [20]</td>
<td>Utilization of regenerative energy</td>
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<tr>
<td>Li and Yang [12]</td>
<td>Utilization of regenerative energy</td>
<td>Stochastic expected value model</td>
<td>Genetic algorithm</td>
<td>Objective 1 improves 8.86% and objective 2 reduces 3.22% in Beijing Yizhuang Line</td>
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<tr>
<td>Yang et al. [21]</td>
<td>1Utilization of regenerative energy 2Passenger waiting time</td>
<td>Integer programming model</td>
<td>Genetic algorithm</td>
<td>Objective 1 improves 21.9% and objective 2 reduces 4.3% in Beijing Yizhuang Line</td>
</tr>
<tr>
<td>Zhao et al. [22]</td>
<td>1Overlapping time 2Total passenger time</td>
<td>Integer programming model</td>
<td>Simulated annealing algorithm</td>
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</tr>
<tr>
<td>Yang et al. [24]</td>
<td>Utilization of regenerative energy</td>
<td>Nonlinear integer programming model</td>
<td>Simulated annealing algorithm</td>
<td>Energy saving of 6.97% in Beijing Yizhuang Line</td>
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</table>

It is noted that the timetable optimization method does not work for urban rail transit systems without regenerative braking. In addition, it can achieve better energy saving performance for short sections than long sections (i.e., with a longer cruising phase).

From the above reviews, we conclude that the studies focusing on improving the utilization of regenerative energy by timetable optimization method have attracted more attention in recent years. The metrics about energy saving in these studies include overlapping time, peak power consumption and utilization of regenerative energy. Increasing the overlapping time and limiting the peak of power consumption just denote improving the utilization of regenerative energy. However, it is difficult to find explicitly mathematical relationships (e.g., linear) among them. A set of recent publications on timetable optimization are summarized in Table I.

IV. ENERGY-EFFICIENT DRIVING

In this section, we introduce the energy-efficient driving method. Firstly, the energy-efficient driving problem is described in detail. Then a comprehensive review of the literature is given.

According to literature [25], the minimum energy consumption of traction system is uniquely determined by the running time as shown in Fig. 8. And there exists a speed profile with minimum energy consumption by a given running time.
Energy-efficient driving method aims at finding the optimal speed profile to minimize the tractive energy consumption.

A. Problem Description

According to the optimal train control theory [26]–[31], the energy-efficient speed profile of trains at each section consists of maximum acceleration, cruising, coasting, and maximum braking. However, for trains with short travel distance (generally less than 5000 m), the energy-efficient speed profile does not contain cruising phase [25], [32]–[34]. As shown in Fig. 9, with the given running time, line conditions, vehicle performances, etc., a set of train’s speed profiles between two successive stations satisfy the operation constraints. The purpose of energy-efficient driving is to find a speed profile with minimum energy consumption. In urban rail transit, the distance between two successive stations is generally very short [15]. Take the Beijing Metro Yizhuang Line as an example, the longest distance between successive stations is 2631 m (from Songjiazhuang to Xiaocunqiao), and the shortest distance is 993 m (from Yizhuangqiao to Wenhua yuan). Therefore, the energy-efficient speed profile of urban rail trains generally does not contain the cruising phase.

Literature on the energy-efficient driving method can date back to 1960s. In 1968, Ishikawa [35] firstly proposed the optimal control model to determine the speed profile. The proposed model can apply to both urban rail transit systems and general railway systems. In 1980, Milroy [36] reformulated the problem as follows:

\[
\begin{align*}
\min \quad & F(u, v) = \int_0^T u_+(t)v(t)dt \\
\text{s.t.} \quad & v'(t) = u(t) - r|v(t)| \\
& v(0) = v(T) = 0 \\
& \int_0^T v(t)dt = L \\
& |u(t)| \leq 1
\end{align*}
\]

(3)

which lays the foundation of the optimal train control theory. Although Milroy applied the model to general railway systems, Howlett et al. [37] proved that it was also suitable for urban rail transit. In this model, the objective function \( F(u, v) \) denotes the tractive energy consumption, \( T \) denotes the given running time by timetable, \( L \) denotes the distance between successive stations, \( u(t) \) denotes the acceleration rate applied to the train, \( v(t) \) denotes the train speed, \( r|v(t)| \) denotes the unit frictional resistance, and \( u_+(t) \) denotes the positive part of \( u(t) \), i.e., \( u_+(t) = \frac{|u(t)| + |u(t)|}{2} \). The first differential constraint equation denotes the final acceleration rate of the train, the second constraint denotes that the speed at starting position and ending position equals zero, the third constraint denotes that the travel distance of the train equals the distance between stations, and the fourth inequality constraint normalizes the acceleration applied to the train. By applying the Pontryagin maximum principle, Milroy obtained a basic speed profile, which was the optimal in his view. The conclusion was supported by Kraft and Schnieder [38], Tyler [39], and Kautsky et al. [40].

B. Literature Overview

Over the decades, further research has been done on energy-efficient driving problem. The methods in the literature are mainly grouped into two categories: analytical algorithm and numerical algorithm [41]. Analytical algorithm requires good properties of the objective function, such that researchers have to simplify some conditions when modeling. Numerical algorithm has no requirement for the objective function. However, analytical algorithm can obtain the optimal solution exactly, even if the process is complicated. For the numerical algorithm, there is a trade-off involved between the accuracy and computational efficiency. Generally, the computation speed is low and it sometimes can only find the local optimal solution. But the accuracy can be guaranteed when using some numerical solvers [42], [43] with sufficient computation time. The premise is that the energy-efficient driving problem is formulated as a mixed integer linear programming model with some approximations.

1) Analytical Algorithm: As mentioned above, the energy-efficient driving method was based on the optimal control theory firstly. So the problem was formulated as continuous optimal control models at the beginning for simplicity. For example, Kokotovic and Singh [44] proposed a nonlinear second-order model to minimize electrical energy consumption. Asnis et al. [45] assumed that the acceleration rate was a continuous control variable with uniform bounds, and used the Pontryagin maximum principle to find the necessary conditions on an optimal speed profile. For seeking a more strict mathematical testimony, Howlett [46], [47] proved that the problem can be formulated in an appropriate function space. They concluded that an optimal speed profile exists and that the speed profile must satisfy a Pontryagin type criterion. Then in 1990, Howlett [48] formulated the energy-efficient driving problem as a finite dimensional constrained optimization model. They used the Pontryagin principle to find the nature of the optimal strategy and to determine the precise optimal strategy. This work is based on the optimization model proposed by Milroy [36].

However, during the periods from 1960s to 1990s, the driving input in most real-world locomotives is discrete, and braking and traction forces cannot be varied continuously. For convenience in practical use, the study on discrete control model to
solve the energy-efficient driving was being concerned, which was mainly done by the Scheduling and Control Group of the University of South Australia. For example, Howlett et al. [37] outlined the theoretical basis with discrete control model for Metromiser system and Long-haul fuel conservation system, which provided driving advice for metros and long-haul rails, respectively. The Metromiser system has been successfully applied to urban rail transits in Australia, Melbourne, Toronto, etc. Howlett and Leizarowitz [49] formulated an algebraic equation rather than a differential equation, from the Euler-Lagrange equation for certain intervals. This algebraic equation was useful for the structure of optimal control scenarios which were composed of segments with pure control and chattering control. Furthermore, the energy-efficient driving problem with a discrete input was studied for a track without varying gradients and speed limits by Cheng and Howlett [50], [51]. They showed that the optimal speed profile depended on three critical values of a prescribed speed sequence of fuel supply rates. Then the problem with speed limits was solved by Pudney and Howlett [52]. They proved that on intervals of track where

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<td>Pontryagin principle</td>
<td>Lay the foundation of the optimal train control theory</td>
<td>Method validation by theory proofs</td>
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<tr>
<td>Howlett [48]</td>
<td>Continuous</td>
<td>Pontryagin principle</td>
<td>(^1) Obtain a complete determination of the solution</td>
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<td>Cheng and Howlett [51]</td>
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<td>Howlett and Leizarowitz [49]</td>
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<td>Formulate a two-level hierarchical model</td>
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<td>Chang and Sim [71]</td>
<td>Continuous</td>
<td>Genetic algorithm</td>
<td>Consider schedules, passenger loads, track voltages</td>
<td>Energy saving of 30% in simulations at laboratory</td>
</tr>
<tr>
<td>Wong and Ho [72]</td>
<td>Continuous</td>
<td>(^1) Direct search methods, (^2) Heuristic search methods</td>
<td>Make a comparison of different search methods</td>
<td>Energy saving of 5-6% in simulations at laboratory</td>
</tr>
<tr>
<td>Chuangetal. [75]</td>
<td>Discrete</td>
<td>Artificial neural network</td>
<td>Minimize both energy consumption and traveling time</td>
<td>Energy saving of 20-30% in Kaohsiung Metro Orange Line</td>
</tr>
</tbody>
</table>
In real-world systems, reducing the maximum speed and increasing coasting are two efficient means of saving energy. In the Sao Paulo system the most energy-efficient driving profile consisted in reducing the maximum speed at the expense of increasing the acceleration rates [70]. Different methods to determine the optimal coasting points and the associated speed profiles have been studied in the literature [71]–[75]. For
example, Chang and Sim [71] applied the genetic algorithm to generate an optimal coast control with predetermined number of coasting points. Wong and Ho [72] presented the application of both direct and heuristic search methods to find the coasting points for a section run with specified running time. Chuang [75] formulated the energy-efficient driving and integrated optimization methods respectively for the optimization of both direct and heuristic search methods to find the coasting points for a section run with specified running time. Chuang [75] developed an artificial neural network to determine the optimal coasting points for a section run with specified running time. Chuang [75] of both direct and heuristic search methods to find the coasting points for a section run with specified running time. Chuang [75] of both direct and heuristic search methods to find the coasting points for a section run with specified running time.

A set of recent publications on energy-efficient driving method are summarized in Table II.

V. INTEGRATED OPTIMIZATION METHOD

Energy-efficient driving only focuses on optimizing the speed profile between adjacent stations for single train. It always ignores the regenerative energy transmitting among multiple trains. Hence the obtained energy-efficient speed profile is only optimal for single train but not for multiple trains. Timetable optimization synchronizes the actions of multiple trains to maximize the utilization of regenerative energy, but it usually assumes the speed profile as a constant parameter. The tractive energy consumption is not reduced by the obtained optimal timetable. Therefore, in recent years, a few researchers studied the integrated optimization method (see Fig. 10) jointly optimizing the timetable and speed profile to maximize the utilization of regenerative energy as well as to minimize the tractive energy consumption. The flow chart of integrated optimization method is shown in Fig. 11. The studies achieved better energy-saving performance than only using timetable optimization method or energy-efficient driving method alone.

Wong and Ho [76] described the regulation and coordination of multi-trains operation for real-time applications with mixed dwell time and running time control. The objectives are energy consumption and service quality, and they used the dynamic programming approach to find the solution. Bocharnikov et al. [34] presented a single train speed profile optimization model considering both tractive energy consumption and utilization of regenerative energy. Furthermore, the authors performed a multi-train simulation to estimate the benefits and effects of the optimal speed profile on minimizing the net energy consumption. Ding et al. [77] formulated the energy-efficient train operation problem as a two-level optimization model and designed a genetic algorithm to search for the optimal solution. The first level determined the appropriate coasting point of section run for trains, and the second level arranged the travel time for each section to minimize the tractive energy consumption. Su et al. [78] proposed a cooperative train control model to reduce the energy consumption, and designed a numerical algorithm to obtain the optimal driving strategy with a given trip time, in which the variable traction forces, speed limits, and gradients are considered. Then Su et al. [79] developed the model and designed a bisection method to solve the optimal timetable and speed profile. The results showed that the developed model can save 2.4% of energy for one trip in comparison with the energy-efficient driving method [25]. Yang et al. [80] developed an integrated optimization method to reduce the total energy consumption and total travel time, they determined the timetable and speed profile by finding the optimal dwell time at stations and maximum train speed at sections. The method can reduce total energy consumption by 7.31% in comparison with the current operation strategy in Beijing Metro Yizhuang Line.

In literature [81], Li and Lo proposed an integer programming model to determine the timetable and speed profile with the minimum total net energy consumption, which is the difference between the tractive energy consumption and the utilization of regenerative energy. They made a comparison among timetable optimization method [11], energy-efficient driving method [25] and the integrated optimization method on the net energy consumption. The results showed that when the headway was 90 s, the integrated optimization method can reduce the total energy consumption by 21.17% compared to the timetable optimization method, and 6.35% compared to the energy-efficient driving method. The results of these three methods are all based on the real-world operation data from Beijing Metro Yizhuang Line. The detailed description of this line is provided in the APPENDIX. The comparisons between the original operation strategy in Yizhuang Line and the strategies optimized by timetable optimization method, energy-efficient driving method and integrated optimization method respectively are summarized in Table III to show the energy saving rates with different optimization methods more clearly. Then Li and Lo [82] developed a dynamic integrated optimization approach with adaptive cycle time based on the passenger demand. The results showed that the proposed dynamic approach can reduce the net energy consumption by 7.86% compared with the static integrated optimization approach [81].

Because of the global optimality on energy conservation and its good energy-saving performance, it is concluded that the integrated optimization method will be a tendency in future research. Most objective functions of the optimization model in the literature were formulated using mechanical energy equations. However, in real-world urban rail transit systems, the substations, trains, overhead contact lines, tracks and others construct an integrated power network. The substations are power sources and trains are loads, which means we should solve the equivalent circuit equations to calculate the energy consumption more accurately. Moreover, train mass is a variable in real-world systems because of uncertain passenger mass.
flow in space and time, but for simplicity, it is assumed as a constant parameter in most of the existing studies. In addition, the optimization model becomes more complicated considering more real-world conditions, such as varying gradients, arbitrary speed limits, and different radius curves. For obtaining the optimal solution, it always takes a long computation time. Reducing the computation time to a reasonable level will be very meaningful to satisfy online optimization demand. Some other future research directions are to extend the integrated optimization method to consider the variable acceleration rate, deceleration rate, headway, running resistance, distance-based energy transmission losses factor, and stochastic departure delay for trains at busy stations for making more realistic calculations of energy consumption.

VI. CONCLUSION

The main contribution of this paper is to give a comprehensive survey on train energy-efficient operation strategies in urban rail transit systems. The operation strategies in the literature are grouped into timetable optimization, energy-efficient driving and integrated optimization method. The current literature on these three methods was reviewed. It was concluded that the integrated optimization method had become a new tendency and ought to be paid more attention in future research.

As already mentioned, a number of assumptions are made in the existing integrated optimization methods for simplifying the model formulation and solution algorithm. For example, the train mass is assumed as a constant parameter; the energy conversion efficiency is considered as a deterministic parameter; and the calculation of energy consumption is simplified by using mechanical energy equations but not modeling the power network. These assumptions should be explicitly tackled in future research.

There are many same problems between urban rail transit and general railway, and the studies in many areas of urban rail transit get the idea from the similar studies of general railway. However, for the energy-efficient operation, researchers have achieved more advanced methods and better results in urban rail transit, which can inspire the research on energy saving for general railway.

In addition, most methods in the literature were proved to be efficient by numerical examples, but few of them were tested in real-world systems. In laboratory simulations, trains always perform the given speed profile strictly, and arrive at each station on time according to the timetable exactly. However, in practice, trains may have some small deviations. These small deviations do not affect the normal operations, but they have some influences on the estimations of energy consumption. Therefore, more empirical studies should be conducted to test its efficiency in practical operations of urban rail transit systems.

APPENDIX

The description of the Beijing Metro Yizhuang Line is as follows. We obtain the current operation data from the Beijing Mass Transit Railway Operation Corporation Limited [83].

The Beijing Metro Yizhuang Line links the downtown of Beijing and the Yizhuang Economic Development Zone, which covers a length of 22.73 km and consists of 14 stations from Songjiazhuang station to Yizhuang station (see Fig. 12).

The length and current running time of each section are provided in Table IV. The minimum headway and maximum speed limited by the signaling system are 90 s and 80 km/h. The operated train is vehicle SFM13. The empty vehicle mass is 199,000 kg. The maximum tractive and braking forces are 315,000 N and 258,000 N.

REFERENCES


Xin Yang received the B.S. degree from Beijing Jiaotong University, Beijing, China, in 2011, where he is currently working toward the Ph.D. degree with the State Key Laboratory of Rail Traffic Control and Safety.

His current research interests include intelligent transportation systems, optimization theory and its applications in transportation problems, and rail traffic control systems.

Xiang Li received the B.S. degree from Jilin University, Changchun, China, in 2004 and the Ph.D. degree in operations research and cybernetics from Tsinghua University, Beijing, China, in 2008.

He is currently a Professor with Beijing University of Chemical Technology, Beijing. He is the author or coauthor of more than 50 articles on international journals, including Transportation Research Part B; Transportation Research Part C; Information Sciences; European Journal of Operational Research; IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS; IEEE TRANSACTIONS ON FUZZY SYSTEMS; and IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS-PART A, which have been cited more than 500 times on Web of Science and 1000 times on Google Scholar. His research interests include uncertainty theory, fuzzy programming, fuzzy logic, and its applications in sustainable railway transport. He served as the Associate Editor of Information Sciences and an Editorial Board Member of International Journal of General Systems.

Bin Ning (F'14) received the Ph.D. degree from Beijing Jiaotong University, Beijing, China, in 2005.

He is currently a Professor with the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, where he is also the President. He was a Visiting Scholar with Brunel University, Uxbridge, U.K., from 1991 to 1992, and with University of California, Berkeley, from 2002 to 2003. His research interests include intelligent transportation systems, communication-based train control, rail transport systems, system fault-tolerant design, fault diagnosis, system reliability, and safety studies. He is a Fellow of the Institute of Electrical and Electronics Engineers, the Association of International Railway Signaling Engineers, and China Railway Society; the Deputy Director of China Traffic System Engineering Society; and an Associate Editor of IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS and Acta Automatica Sinica.

Tao Tang received the Ph.D. degree from the Chinese Academy of Sciences, Beijing, China, in 1991.

He is the Academic Pacesetter with the National Key Subject Traffic Information Engineering and Control and the Director of the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing. He is also a Specialist with the Beijing Urban Traffic Construction Committee.

His research interests include both high-speed and urban railway train control systems, as well as intelligent control theory.