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Abstract

Robot-assisted surgery is gaining popularity worldwide and there is increasing scientific interest to explore the potential of soft continuum robots for minimally invasive surgery. However, the remote control of soft robots is much more challenging compared with their rigid counterparts. Accurate modeling of manipulator dynamics is vital to remotely control the diverse movement configurations and is particularly important for safe interaction with the operating environment. However, current dynamic models applied to soft manipulator systems are simplistic and empirical, which restricts the full potential of the new soft robots technology. Therefore, this article provides a new insight into the development of a nonlinear dynamic model for a soft continuum manipulator based on a material model. The continuum manipulator used in this study is treated as a composite material and a modified nonlinear Kelvin–Voigt material model is utilized to embody the visco-hyperelastic dynamics of soft silicone. The Lagrangian approach is applied to derive the equation of motion of the manipulator. Simulation and experimental results prove that this material modeling approach sufficiently captures the nonlinear time- and rate-dependent behavior of a soft manipulator. Material model-based closed-loop trajectory control was implemented to further validate the feasibility of the derived model and increase the performance of the overall system.

Keywords: soft continuum robots, dynamics, biologically inspired robot, constitutive law

Introduction

The robotic technology has revolutionized the world by automating tasks, enhancing safety, as well as improving the efficiency in various applications. Technological advancement in robotics for the past few decades has given rise to the interesting and challenging field of soft robotics. Soft robotics technology has a promising future and provides new capabilities in diverse fields such as health care technologies, biomedical systems, and search and rescue systems. It offers flexible mechanism and dexterous mobility, therefore, is ideal for applications that require delicate operation in a congested and unstructured environment. This emerging technology is enabled by advances in soft and compliant materials, improvement of nonlinear modeling and intelligent control, as well as conformable sensors and electronics. Soft robots are also lightweight, have a high power-to-weight ratio, and can be easily fabricated with the advancement in digital fabrication techniques. Numerous prototypes have been designed for realization of soft continuum robotic manipulators based on inspiration from octopus, caterpillar, snake, and elephant trunk. The flexibility and maneuverability of these prototypes demonstrate a strong potential of soft robotics technology.

Soft robotics offers an attractive alternative to current medical robotic technology, particularly for the minimally invasive surgery. Major advantage of this technology over the rigid link manipulator is that a soft manipulator is...
inherently compliant and generates little resistance to compressive forces, therefore, can conform to obstacles.\textsuperscript{11} Hence, the manipulator can safely go through narrow openings and interact with organs without causing any damage to the tissue.

Exploiting the actual capabilities of a soft robotic arm, specifically its dexterity and mechanical performance, requires a comprehensive and numerically stable model of the system for real-time implementation. Formulating the model proves to be very challenging due to the continuous nature, flexible material, and, hence, infinite number of degrees of freedom of the manipulator. The common assumption of the traditional multirigid segment robot does not hold true for soft robots. Efforts in modeling soft robotic manipulators have focused mainly on kinematic modeling,\textsuperscript{9,10,12} which is based on the geometrical shape of the manipulator, whereas the system dynamics is poorly studied, and differences in the dynamic mechanical behavior of different materials are rarely captured. Soft materials such as silicone, Ecoflex, rubber, or gels\textsuperscript{13} follow highly hyperelastic behavior,\textsuperscript{14} which needs to be taken into account into a continuum mechanics model rather than extending the classical elastic beam theory for rigid segments to soft robots. Furthermore, such soft materials exhibit considerable amount of viscous behavior that is demonstrated by terms of heat loss, which results in hysteresis in medium-to-high rate actuation–strain curves of a soft robot arm, as shown in the experiments by Godage\textit{et al.}\textsuperscript{15} For these reasons, lack of a general dynamical model in portraying the complex nonlinear time- and rate-dependent behavior of this type of manipulator constricts the full potential of the technology.

Dynamic modeling of continuum manipulators remains an active area of research. Some of the earliest studies in deriving the dynamic model of a flexible manipulator are presented by Chirikjian\textit{et al.}\textsuperscript{16} for a hyper-redundant manipulator, by Khalil\textit{et al.}\textsuperscript{8} for a serial eel-like robot, and by Matsuno and Sato\textsuperscript{17} for a snake robot. However, these studies are applied to robots that at best have hyper-redundant rigid links but are not fully continuum robots. Tatlicioglu\textit{et al.}\textsuperscript{18,19} formulated the dynamic models for a planar continuum manipulator. Even though the results by Khalil\textit{et al.}\textsuperscript{8} describe the dynamic model for a multisegment manipulator, the model is two-dimensional (2D), which limits its practical application for a physical system. Mochiyama and Suzuki\textsuperscript{20} extended the work of Chirikjian\textsuperscript{16} by providing the Newton–Euler and regressor representation of the model for the three-dimensional (3D) system. However, no experimental validation was presented to assess the performance of each representation.

The studies by Godage\textit{et al.}\textsuperscript{15,21} capture the 3D dynamics of each actuating chamber. The study by Godage\textit{et al.}\textsuperscript{15,21} presents the derivation of the dynamics model of continuum manipulator for underwater application based on the modal approach. In Godage\textit{et al.}\textsuperscript{15,21} empirical fitting of experimental data was used to capture the nonlinearity and viscous behavior of the system. The Bouc–Wen empirical model was employed\textsuperscript{15} to predict the module hysteresis, and its parameters were fitted to the experimental data for a three-pneumatic channel actuator with their particular system of pressure control, without including a material-based model for the rate dependence of the hysteresis or viscous behavior. The predictions by Godage\textit{et al.}\textsuperscript{15,21} were in good agreement with the experimental data, although their applicability was limited to the particular experimental system and rig used in each study (tested in fact for one actuating pressure chamber only) and have no general applicability to the modules of different geometries, made from different materials, and tested under different rates of deformation or pressure combination.

The latest study on the dynamics of soft manipulator is by Gazzola\textit{et al.}\textsuperscript{22} The authors produced a dynamic model based on Cosserat rod model, to capture shear, bending, and stretching of the filament. The model although extensive and demonstrates its applicability in various environments, it lacks the nonlinear time- and rate-dependent behavior of a soft manipulator itself.

The dynamic models described previously investigated a single-pneumatic channel that is structurally different from the actuation mechanism in this study. Suzumori\textit{et al.}\textsuperscript{23,24} introduced the first prototype of pneumatically actuated three-chamber (PTC) soft modules with low operating pressure. However, Suzumori\textit{et al.}\textsuperscript{23} derived the dynamic characteristics of PTC module based on the deflection distance, projected on to the horizontal base plane, which is only valid for a small bending angle. This limits and complicates its application for real-time control. One of the latest studies in dynamic modeling of soft continuum robot can be found in Renda\textit{et al.}\textsuperscript{2} Their approach utilizes the Kelvin–Voigt model to capture the viscoelastic properties of the material used to construct the prototype. However, the assumption of elastic material behavior, whereby the stress–strain relationship is linear, may not accurately capture the true nature of a hyperelastic soft material system. Trivedi\textit{et al.}\textsuperscript{25} developed a nonlinear continuum model for the deformation of a pneumatically actuated soft robot arm and modeled the inextensibility of the reinforced fiber as part of, but carried out only a static solution of their equations and did not proceed to dynamic modeling. The study by Polygerinos\textit{et al.}\textsuperscript{26} assumed hyperelastic behavior of reinforced silicone and presented a good quasi-static analytical model for a single-chamber soft robot actuator. Similar to Trivedi\textit{et al.}\textsuperscript{25} the full structure dynamic model was not derived in Polygerinos\textit{et al.}\textsuperscript{2}

In this article, a novel material-based dynamic model is introduced for a STIFF-FLOP (STIFFness controllable Flexible and Learnable manipulator for surgical OPeration) single module, which is a three-pneumatic chamber actuator, with three fiber-reinforced pneumatic chambers encapsulated within a silicone module. The dynamics of the STIFF-FLOP module have been extensively studied by Sadati\textit{et al.}\textsuperscript{26–28} The study by Sadati\textit{et al.}\textsuperscript{26} used the identification-based solution, where the shape function of the manipulator is produced. Although this model provided good prediction, it has no general applicability to module of different geometries and materials. In Sadati\textit{et al.}\textsuperscript{27} a modified Lagrange polynomial is introduced using Ritz and Ritz–Galerkin methods. This approach proved to improve modeling accuracy; however, the model did not capture the material hysteresis and damping effects of this type of soft manipulator. The study by Sadati\textit{et al.}\textsuperscript{26} focused on geometry deformation in improving the model performance as well providing insight into choosing proper assumptions for modeling this type of manipulator. Furthermore, the model takes into consideration the effects of soft materials in large deformation in modeling of the soft manipulator. The authors employed the
neo-Hookean hyperelastic material model but disregarded the viscous property of the material. Therefore, the model has yet to properly capture the hysteresis effect of the soft material. Hence, there is the need for an improved dynamic model, for the STIFF-FLOP manipulator.

In this article, a novel material-based dynamic model is introduced for a three-pneumatic chamber actuator, with three fiber-reinforced pneumatic chambers encapsulated within a silicone module. The main contribution of this article is on the modeling approach to embody the nonlinear dynamics of a composite soft continuum manipulator based on the constitutive law of material behavior. The Kelvin–Voigt model for viscoelastic behavior (used in Renda et al.) is modified to an equivalent novel model comprising a hyperelastic element in parallel with a non-Newtonian power law visco-elastic type, to accommodate the non-Newtonian visco-hyperelastic behavior of the silicone material, and accurately capture the true dynamic behavior of the system. The authors have previously presented the concept of modified Kelvin–Voigt model in Trivedi et al., in which, however, the model was validated only in simulation of a single-chamber operation. The previous study also omitted the mechanical properties of the reinforced fiber composite layer of the soft continuum module and only focused on the mechanical properties of the silicone body of the module. In this study, the same modified Voigt model is applied for a multichannel operation, together with the inverse rule of mixture iROM in determining the mechanical properties of the composite layer, to further improve the accuracy of the proposed material-based model for the proposed soft continuum module. The single-channel formulation is reformulated in this study and then extended to the formulation for a multichamber system. The model is experimentally validated and applied in a material model-based closed-loop position control case study.

**Design of Prototype**

The flexibility of this manipulator is achieved by adopting the concept of flexible microactuator presented by Suzumori et al. Figure 1 shows the design of the prototype. It was fabricated using Ecoflex™ 0050 silicone and is composed of three equally spaced cylindrical pneumatic chambers disposed at 120° apart in symmetrical radial arrangement. Each pneumatic chamber is reinforced with a thin nylon fiber applied in a tight helix around the chamber to allow for longitudinal expansion, constrain any radial inflation, and maximize the bending upon pressure actuation. All three chambers were molded directly into a single cylindrical unit with Ecoflex 0050 filling the space between the fiber-reinforced chambers. The diameter of each actuating chamber is 8 mm and the diameter of the whole cylindrical unit is 28 mm. The module is lightweight with a mass of 24 g and with an initial length of 45 mm. Further details of the design for this prototype can be found in Fras et al.

**Mathematical Preliminaries**

**Kinematic modeling**

The general and widely used approach to kinematic modeling for a continuum soft tube is based upon the framework for piecewise constant-curvature continuum robots, which enables the kinematics to be decomposed into two mappings as follows:

- **Robot-dependent mapping**—mapping between joint space \( q = \{q_1, q_2, q_3\} \) and configuration space \( \{\kappa, \phi, L\} \)
- **Robot-independent mapping**—mapping between configuration space \( \{\kappa, \phi, L\} \) and task space \( \{x, y, z\} \).

where \( q_i \) describe the length change of each actuator, which in the case of this study are the length change of each pneumatic chamber and the arc parameters \( \{\kappa, \phi, L\} \) are curvature radius, orientation angle, and the arc length, respectively. The robot-dependent mapping describes the spatial orientation and deformation of the manipulator upon actuation according to the following relationship [Eq. (1)]:

\[
L(q) = \frac{L_1 + L_2 + L_3}{3}
\]

\[
\phi(q) = \tan^{-1}\left(\frac{\sqrt{3}(L_2 + L_3 - 2L_1)}{3(L_2 - L_3)}\right)
\]

where \( L_1, L_2, \text{ and } L_3 \) are the length of each channel, and \( d \) is the distance from the center of the module to the center of each channel. The robot-independent mapping describes the coordinate transformation between task space and configuration space and is applicable for a wide range of continuum manipulators. It can be derived based on the conventional Denavit–Hartenberg or by employing the standard rotational and translational matrices as given in Equation (2). The relationship is represented by the homogeneous transformation matrix (HTM), \( T \).
Dynamic modeling

The overall dynamical model of the system is obtained by employing the Lagrangian analysis, similar to the approach used by Della Santina et al. The equation of motion can be obtained by applying the Euler–Lagrange equation formulated in Equation (3):

\[ F = \left( \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} \right) \]  

where \( L \) is the Lagrangian defined as

\[ L = E_k - E_g - E_m. \]  

\( F \) in Equation (3) is the generalized force and \( q \) is the joint space coordinate vector. In Equation (4), \( E_k \) is the kinetic energy, \( E_g \) is the gravitational potential energy, and \( E_m \) is the mechanical energy due to mechanical stresses (elastic and viscous stresses for a viscoelastic material). The basic formulations of \( E_k, E_g, \) and \( E_m \) are explained in the subsequent corresponding subsections for a single- and a three-chamber system.

Energy formulation. Figure 2 depicts the diagram of a three-pneumatic chamber continuum manipulator and its kinematics nomenclature. \( L_o \) is defined as the initial length of the cylindrical module and \( L(t) = L_o + q(t) \) is the length of the module center line at any time, where \( q(t) \) is the module extension after pressure is applied. Taking a similar approach as in Godage et al., the soft module is assumed to be made up of infinitesimal slices of thickness \( L \delta \xi \), where \( L = 0/\kappa \) is the length of the continuum arm along the central axis.

The total kinetic energy of the system is

\[ E_k(q, \dot{q}) = \frac{1}{2} \int_0^1 \left(V^p_{\xi} \right)^T \delta M \dot{V}^p_{\xi} d\xi \]

and the gravitational potential energy of the system is

\[ E_g(q) = M \int_0^1 p^T g d\xi, \]

where \( g = [0 \ 0 \ 9.81]^T \), and \( p^T \) is the positional vector from Equation (2). In Equation (5), the body velocity of the manipulator, \( V^b_{\xi} \in \mathbb{R}^6 \), was formulated as in Equation (7):

\[ V^b_{\xi} = \left[ \begin{array}{c} v^b_{\xi} \\ \omega^b_{\xi} \\ \frac{R^T p}{R^T R} \end{array} \right] = J^b_{\xi}(q) \dot{q}, \]

where \( R \) and \( p \) are the rotational and translational matrix of the HTM \( T \) as in Equation (2), respectively, \( v^b_{\xi} \in \mathbb{R}^3 \) is the linear velocity component of the center of mass, \( \omega^b_{\xi} \in \mathbb{R}^3 \) is the instantaneous angular velocity, and \( J^b_{\xi} \in \mathbb{R}^{6 \times 3} \) is the body Jacobian matrix. \( M \) is a generalized mass matrix defined as in Equation (8):

\[ M(q) = \int_0^1 \left(J^b_{\xi} \right)^T \delta M \left(J^b_{\xi} \right) d\xi, \]

where \( \delta M = M \cdot \text{diag} \{ 1, 1, 1, 1, R^2, 1, R^3 \} \) is the slice inertia matrix and \( R \) is the radius of the slice. The manipulator is assumed to have a uniform linear density \( \rho = \frac{M}{L} \), where \( M \) is the total mass of the entire manipulator.

The mechanical stress-related energy takes into account the axial strain energy due to extension or compression of a channel and the shear energy present during bending of the module as in Equation (9):

\[ E_m = E_{\text{strain}} + E_{\text{shear}}, \]

where \( \sigma_i \) is the chamber tensile stress, \( \epsilon_i \) is the chamber axial strain, and \( \nu_{si} \) is the chamber volume, for each of the three chambers \( i = 1 \) to \( 3 \); \( \tau \) is the module shear stress and \( \nu_s \) is the
module volume. The tensile and shear stresses are modeled incorporating the material model described in the following section.

Material Model

The novel material models presented in this section are incorporated in the mechanical energy terms of Equation (4) describing the module deformation energy and viscous losses. The three-pneumatic channel module modeled in this study and presented in Figure 3 is composed of multiple material parts. More specifically, each pneumatic channel has a soft silicone (Ecoflex 0050) lumina of internal radius $r_i$ and silicone wall thickness $h_i$ (without any fibers), surrounded by a fiber-reinforced silicone composite layer of wall thickness $h_c$ to an outer reinforced channel radius, $r$. Soft silicone then fills the module space between the reinforced channels. A novel non-Newtonian visco-hyperelastic constitutive model is considered for the soft silicone.

The circular fiber reinforcement prevents radial expansion in each pneumatic channel and allows only for channel elongation upon actuation under gas pressure. Considering axial tensile stress applied to each pneumatically actuated elongation upon actuation under gas pressure. The visco-hyperelastic constitutive model proposed for the soft silicone in this study is described by a system of a non-Newtonian viscous dashpot and a hyperelastic spring in parallel, which is equivalent to the simplified Kelvin–Voigt model so that the stress, $\sigma$, is given by Equation (15)

$$\sigma = \sigma_{he} + \sigma_v,$$

where $\sigma_{he}$ and $\sigma_v$ are the hyperelastic and viscous stress component, respectively.

The standard approach to describe the strain energy of the hyperelastic material is by applying a polynomial energy model. For this study, Yeoh’s reduced polynomial hyperelastic model has been chosen to describe the hyperelastic energy potential:

$$U_{hyp}(q) = \sum_{i=0}^{n} C_i (I_1(q)) - 3/2,$$

where $I_1$ is the first deviatoric strain invariant. The values of $C_i$ obtained from fitting experimental mechanical test data of tensile and shear tests using Ecoflex 0050 are tabulated in Table 1. The strain invariant $I_1$ depends on the type of applied loading and is discussed in the next section.

![FIG. 3. Geometric parameters of the reinforced pressure chamber.](Image)

**Table 1. Geometric and Material Model Parameters (Uniaxial Tensile and Shear Tests Were Conducted for Ecoflex 0050 in the Strain Range of 0–400% and at Strain Rates of 1–1000 mm/min)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_s$</td>
<td>24 g</td>
<td>$\eta_0$</td>
<td>22,263.8 Pa s$^{0.3}$</td>
</tr>
<tr>
<td>$L_0$</td>
<td>45 mm</td>
<td>$n$</td>
<td>0.3</td>
</tr>
<tr>
<td>$R$</td>
<td>14 mm</td>
<td>$C_{10, e}$</td>
<td>12,563 J/m$^3$</td>
</tr>
<tr>
<td>$r$</td>
<td>4 mm</td>
<td>$C_{20, e}$</td>
<td>-67,784 J/m$^3$</td>
</tr>
<tr>
<td>$r_s$</td>
<td>2.25 mm</td>
<td>$C_{30, e}$</td>
<td>2.7385 J/m$^3$</td>
</tr>
<tr>
<td>$r_i$</td>
<td>1.15 mm</td>
<td>$C_{10, s}$</td>
<td>12,563 J/m$^3$</td>
</tr>
<tr>
<td>$V_{m0,1}$</td>
<td>0.115</td>
<td>$C_{20, s}$</td>
<td>-67,784 J/m$^3$</td>
</tr>
<tr>
<td>$V_{m1,1}$</td>
<td>0.103</td>
<td>$C_{30, s}$</td>
<td>2.7385 J/m$^3$</td>
</tr>
<tr>
<td>$V_{m0,1}$</td>
<td>0.095</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The viscous part of the visco-hyperelastic material model is governed by the following equation:

$$\sigma_v = \eta \frac{d\varepsilon(t)}{dt},$$

(17)

where $\eta$ is the power law viscosity$^{40}$ defined as

$$\eta(n) = \eta_o \left( \frac{d\varepsilon(t)}{dt} \right)^{n-1},$$

(18)

where $\eta_o$ is the consistency of the material and $n$ is the power law index for a non-Newtonian fluid. If $n$ is $<1$, the power law predicts that the effective viscosity would decrease with increasing deformation rate, which is usually the case for polymeric materials. Thus, the viscous energy potential of the visco-hyperelastic constitutive model is described by Equation (19):

$$U_v(q, \dot{q}) = \frac{1}{2} \eta_o \left( \frac{d\varepsilon(t)}{dt} \right)^{n-1} \varepsilon(t).$$

(19)

**System Dynamics**

In this section, the mechanical potential and viscous energy due to extension and bending will be derived based on the axial strain and shear energy formulations of the constitutive model introduced in the previous section, leading to the derivation of the overall dynamic model for the single-chamber actuation and the multiple-chamber actuation.

**One-chamber actuation**

The axial strain energy and shear energy components of Equation (9) will be derived for one-chamber actuation (chamber $i$). The overall axial strain energy is given as the sum of mechanical potential (hyperelastic) and viscous energy due to extension of the actuating chamber $i$ as follows:

$$E_{i,\text{strain}} = E_{i,\text{strain,he}} + E_{i,\text{strain,v}}$$

$$E_{i,\text{strain,he}} = v_s \left[ C_{10,e} (\dot{I}_{1,\text{strain}},i - 3) + C_{20,e} (\dot{I}_{1,\text{strain}},i - 3)^2 \right. + \left. C_{30,e} (\dot{I}_{1,\text{strain}},i - 3)^3 \right]$$

$$E_{i,\text{strain,v}} = v_s \left[ \frac{1}{2} \frac{\eta_o}{L_0} \left( \frac{q_i}{L_0} \right)^{n-1} \dot{q}_i \dot{q}_i \right].$$

(20)

For the case of axial strain under the constant volume hypothesis, the first deviatoric strain invariant for chamber $i$, $I_{1,\text{strain},i}$, is described as follows$^{28,38}$:

$$I_{1,\text{strain},i} = \left( 1 + \frac{q_i}{L_0} \right)^2 + 2 \left( 1 + \frac{q_i}{L_0} \right)^{-1} \dot{q}_i \dot{q}_i.$$  

(21)

One-chamber actuation results in the bending of a PTC soft module, which also involves shear stresses as illustrated in Figure 4. The hyperelastic mechanical shear energy depends on the first deviatoric invariant for shear defined as follows$^{41}$:

$$I_{1,\text{shear}} = 3 + \gamma^2,$$

(22)

where $\gamma$ is the shear angle, which is half of the bending angle $\theta : \gamma = \frac{\theta}{2}$. Thus, the mechanical shear energy due to the bending of the soft module can be represented as follows:

$$E_{\text{shear}} = E_{\text{shear,he}} + E_{\text{shear,v}}$$

$$E_{\text{shear,he}} = v_s \left[ C_{10,s} \left( \frac{\theta}{2} \right)^2 + C_{20,s} \left( \frac{\theta}{2} \right)^2 + C_{30,s} \left( \frac{\theta}{2} \right)^3 \right]$$

$$E_{\text{shear,v}} = v_s \left[ \frac{\eta_o}{2} \left( \frac{\theta}{2} \right)^{n-1} \dot{\theta} \dot{\theta} \right].$$

(23)

where $E_{\text{shear,he}}$ and $E_{\text{shear,v}}$ are the mechanical shear energy terms derived based on the hyperelastic and viscous properties of the module, respectively. The bending angle $\theta$ is derived from the joint space representation using Equation (1), and for a single-chamber actuation, the deformation length of the other two chambers is considered zero.

Based on these energy formulations, the Euler–Lagrange Equation (3) can be rewritten as

$$F = \left( \frac{d}{dt} \frac{\partial E_s}{\partial q} \right) \left( \frac{\partial E_s}{\partial q} \right) - \left( \frac{d}{dt} \frac{\partial E_s}{\partial \dot{q}} \right) \frac{\partial E_s}{\partial \dot{q}}$$

$$+ \frac{\partial (E_{\text{strain}} + E_{\text{shear}} + E_g)}{\partial \dot{q}} + \frac{\partial (E_{\text{strain}} + E_{\text{shear}} + E_g)}{\partial \ddot{q}}.$$

(24)

The newly derived dynamics of the single-chamber system based on material modeling is then represented by the following equation:

$$F_i = M \ddot{q}_i + D_1(q_i, \dot{q}_i) \dot{q}_i + D_2(q_i, \dot{q}_i) + G(q_i),$$

(25)
where \( M \) is mass, terms containing \( D_1 \) and \( D_2 \) are associated with damping, and \( G \) is the sum of the elastic and gravitational forces. By considering the viscous term of Equations (20) and (23) for \( D_1 \) and \( D_2 \), these terms are formulated as follows:

\[
D_1 \ddot{q}_i = \left[ -\eta_0 n(n-1)q_i \dot{q}_i + \frac{v_{ni}}{L_0 + 1} + \frac{v_s}{3n+1} \right] \ddot{q}_i, \tag{26}
\]

\[
D_2 = -\eta_0 n \left[ \frac{v_{ni}}{L_0 + 1} + \frac{v_s}{3n+1} \right],
\]

\[
G = \frac{3m_g d^2}{4q_i^2} \left[ -L_0 \sin \left( \frac{2q_i}{3d} \right) + \frac{2d}{3d} \left( L_0 + \frac{q_i}{3} \right) \cos \left( \frac{2q_i}{3d} \right) \right]
+ v_{ni} \left( \frac{2}{L_0} \left( 1 + \frac{q_i}{L_0} \right) - \frac{2}{L_0 \left( 1 + \frac{q_i}{L_0} \right)} \right) \tag{27}
\]

\[
\left[ C_{10,e} + C_{20,e} \left( \Bar{I}_{1(strain,i)} - 3 \right) + C_{30,e} \left( \Bar{I}_{1(strain,i)} - 3 \right)^2 \right] + v_s \left( C_{10,s} \frac{2q_i}{9d^2} + C_{20,s} \frac{4q_i^3}{81d^2} + C_{30,s} \frac{2q_i^5}{243d^6} \right). \tag{28}
\]

**General three-chamber actuation**

The three-chamber actuation model caters for actuation of more than one pneumatic chamber. Similar to the single-chamber system, the mechanical potential and viscous energy due to the extension and bending of each of the three chambers are as presented by Equations (20) and (23). If more than one chamber is under actuation, the sum of the mechanical potential and viscous strain energy from the extension of each channel is considered, whereas the shear energy is considered for the whole module with the bending angle depending on the length change of all three chambers as formulated in Equation (1); this is described by the following equation:

\[
E_{strain} = \sum_{i=1}^{3} \left( E_{i,strain,he} + E_{i,strain,v} \right) + E_{shear,he} + E_{shear,v} \tag{29}
\]

The overall equation of motion is as in Equation (30) and is obtained by employing the Lagrange–Euler formulation as previously discussed for a single-chamber system.

\[
F = M(q) \ddot{q} + D_1(q, \dot{q}) \ddot{q} + D_2(q, \dot{q}) + G(q) \tag{30}
\]

where \( M \in \mathbb{R}^{3 \times 3} \), \( D_1 \in \mathbb{R}^3 \), \( D_2 \in \mathbb{R}^3 \), and \( G \in \mathbb{R}^3 \) are as explained previously and \( F \) is the input force vector in joint space. \( D_1, D_2 \), and \( G \) are as follows:

\[
D_1 = \begin{bmatrix} D_{1,1} & D_{1,2} & D_{1,3} \end{bmatrix}^T \tag{31}
\]

\[
D_2 = \begin{bmatrix} D_{2,1} & D_{2,2} & D_{2,3} \end{bmatrix}^T \tag{32}
\]

\[
G = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix}^T \tag{33}
\]

The entries for vectors \( D_1, D_2 \), and \( G \) are defined as

\[
D_{i,j} \ddot{q}_i + D_{2,i} = \frac{d}{dt} \frac{\partial (E_{i,strain,he} + E_{shear,he} + E_g)}{\partial q_i}. \tag{34}
\]

\[
G_i = \frac{\partial (E_{i,strain,he} + E_{shear,he} + E_g)}{\partial q_i}. \tag{35}
\]

The dynamic problem of this study based mainly on Equation (30) is solved according to the backward Euler numerical technique to avoid any numerical instabilities observed at pressure step changes when the forward Euler technique was used initially. A time step of 0.1 ms was found suitable for all simulations reported in the case studies and the control case study described in the following section.

**Implementation and Experimental Results**

**Implementation**

Figure 5 shows the experimental platform with real-time capability in collecting the required data for this study. The software framework used for coding and real-time implementation is the robot operating system and the RoNeX II board\(^4\)\(^2\) by Shadow Robot is used as the central platform interface between the host PC and hardware component as shown in Figure 5. Each of the pressure lines to the chambers is connected to two solenoid valves and a constant pressure transducer connected at the air inlet of each chamber. A dead zone was introduced at the low-level pressure controller to reduce the undesired frequent switching of the valve. A Vicon motion capture system\(^4\)\(^3\) is used to capture and analyze the bending and elongation of the module at each level of actuating pressure. Three cameras are used for motion analysis purposes.

**Results: system under single-chamber actuation**

Open-loop numerical simulation and experimental testing were carried out to validate the accuracy and investigate the performance of the proposed model. Geometric parameters and properties of the proposed material model are listed in Table 1. The input pressure for the validation test was a step-on-step pressure command, as shown in Figure 6a, from 0.2 to 1 bar pressure with 0.2 bar increments. A low-level closed-loop control system, described above, was constructed to
maintain the pressure at its desired level. A block diagram of the complete setup is as displayed in Figure 5. Chambers 1, 2, and 3 were actuated independently and the average change in length data during the actuations is compared against the model output as can be seen in Figure 6b. The error bars represent the standard deviation in changes in length between these runs.

As depicted in Figure 6b, the model emulates the behavior of the module quite well for pressure $P > 0.4$ bar. The discrepancy at lower range is due to several factors such as the initial gas filled-in phase where the air under low injected pressure takes some time to occupy the space within the pressure chamber. Most importantly, it can be seen in Figure 6b that the novel dynamic visco-hyperelastic continuum model captures well the strain hysteresis at all steps of pressure reduction, as also evidenced by the experimental data. Other factors contributed to the discrepancies are due to fabrication differences between the three pneumatic channels as well as random distortions during actuations.

**Results: system under three-chamber actuation**

The performance of the model under actuation of more than one pneumatic chamber was tested to validate it in the presence of forces applied by more than one actuated chamber. A step input pressure command at $t = 10$ s was applied to each actuating chamber as follows:

$$P = \begin{bmatrix} a \\ 0.4 \\ 0.5 \end{bmatrix},$$

where $a$ is a dependent input command whereby the step input starts from its previous steady-state value as shown in Figure 7a. This experiment is important in observing the response of the model in driving the tip position from one point to another. The input pressure $a$ ranges from 0 to 1 bar with 0.2 bar increments as shown in Figure 7a and the comparison of the change in length response between model prediction and experimental data for all three chambers is illustrated in Figure 7b–d.

Slight discrepancies were observed in the change in length between the model predictions and the experimental data. Nevertheless, the error is small up to 2.6 mm for all three chambers and can be explained by fabrication differences between the three chambers and random defects. Similar observation can be made in terms of the bending angle whereby there is a slight error within the range of $1^\circ$ to $5^\circ$ between the model and the physical system. These results prove that the model is sufficiently good at predicting the changes in length and the bending angle at different values of pressure. The percentage of error between experimental data and simulation is as shown in Figure 7f, in which the percentage of error is plotted against the varying pressure of chamber 1. Error for the individual chamber length is calculated as the difference between the length of the model and experiments divided by the experimental length of the

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**FIG. 5.** Experimental platform: (a) schematics and (b) hardware setup. Color images are available online.

**FIG. 6.** Simulation and experimental result of open-loop single-chamber actuation test under pressure step change: (a) input pressure and (b) average change in module length.
manipulator as a reference length. As depicted in this figure, the error is <5% for pressure >0.4 bar, which is consistent with what was also observed for single-chamber actuation. This finding is an improvement from other STIFF-FLOP models by Sadati et al.,27 where an average of 6% (≈4 mm) mean error was reported. Although the error is small, 3–4 mm discrepancies can be detrimental especially for medical application. Furthermore, this error needs to be rectified before extending this model to a multisegment system, since this error will be transmitted to the preceding sections.

Dynamic testing at different rates. Dynamic testing was carried out that comprises the change of actuation pressure of one chamber at different rates, to observe any viscoelastic effect in both experiment and simulations, which should be more intensive at higher rates of pressure change. These tests were a follow-on of the test shown in Figure 7, consisting of constant pressure in chambers 2 and 3 at 0.4 and 0.5 bar, respectively, and a stepwise change of pressure in chamber 1 from 0 to 1.2 bar and back to 0 bar at different rates in consecutive cycles from 0.04 to 0.666 bar/s. The extension of each chamber and the extension of the module central line were measured. Figure 8 shows a comparison between the experimental and simulation results of the extension of the module central line versus the pressure of chamber 1 for different rates of pressure change in this chamber. It can clearly be seen that higher rates of pressure change cause more severe hysteresis, which is due to the viscous effects associated with the corresponding higher rates of module deformation, and this is captured by both the experimental and simulation results. Figure 8 shows excellent agreement between the model predictions and the experimental data and demonstrates the need and usefulness of the novel visco-hyperelastic material model in the dynamic continuum model of this study to model soft robot actuation at different rates. Figure 8 also shows the simulation results if the viscous term is missing from the model. The viscous term contributes to the time- and rate-dependent effects of the model.

Control implementation

A model-based closed-loop control as shown in Figure 9 was implemented to demonstrate the capability of the model to use the actual pressure information in estimating the length of each chamber to track a desired location within the
FIG. 8. Multichamber actuation dynamic tests and simulation results. Input pressure to chamber 2 and chamber 3 is kept constant at 0.4 and 0.5 bar, respectively. Input to chamber 1 is varied in 12 steps from 0 to 1.2 bar and back to 0 in consecutive cycles at different rates as follows: (a) 0.04 bar/s, (b) 0.05 bar/s, (c) 0.066 bar/s, (d) 0.1 bar/s, (e) 0.2 bar/s, (f) 0.4 bar/s, and (g) 0.66 bar/s.

FIG. 9. Model-based feedback control.
reachable workspace. The control algorithm was run on an Intel® Core™ i5 3230M @ 2.6 GHz processor. The model equation was discretized to allow for the numerical evaluation in producing the length information. The tested input trajectory to the system was an ellipse, consisting of 36 time steps of trajectory as shown in Figure 10. A 2D $x$–$z$ plot was selected to give a better visualization of the actual position with respect to the goal position compared with a 3D $x$–$y$–$z$ plot. An inverse Jacobian as described by Jones et al.\(^{44}\) is used to convert the Cartesian coordinates into the desired module length, bending, and orientation angle $\{L, \theta, \varphi\}$ to achieve the set trajectory that is described by the required length $\{L_{1d}, L_{2d}, L_{3d}\}$ of each channel according to the inverse relations. The controller is driven by the error signal between the desired trajectory and the $\{L, \theta, \varphi\}$ values provided by the model. The output of the controller is fed back to the model block to update the current length of each chamber. Both static and dynamic tests were carried out to test the feasibility of the model in both conditions. Figure 10 shows the model capability in tracing the trajectory in both the dynamic and static conditions and Figure 11 shows the tip position in the $x$, $y$, and $z$-axis. For the static case, a mean error of 1.3 mm was observed, which is an improvement from the previous open loop test (mean error 3.1 mm). The mean error for the dynamic test was 1.8 mm. We can conclude that our proposed novel visco-hyperelastic material-based continuum mechanics dynamic model is reliable in tracking the desired trajectory.

**Limitations**

The model prediction does not consider the effect of interaction with the environment. The assumption of deformation under constant curvature might be invalid if an external load is considered. Further investigation and improvement of the model are needed for force/interaction control applications using the proposed model. Another notable limitation is the significant deviation at lower pressure value, which makes control and tracking around zero backbone curvature a challenging task. Similar observation has been discussed by Marchese et al.,\(^{45}\) suggesting the contribution of nonlinear

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**FIG. 10.** Comparison of trajectory tracking, for both static and dynamic tests. The data are the results of model-based feedback control using visco-hyperelastic material-based dynamic model. Color images are available online.

**FIG. 11.** Tip position in $x$, $y$, and $z$-axis for both (a) static case and (b) dynamic case.
and bistable deformations of the actuation channel at lower pressures.

**Conclusion**

The review of the state-of-the-art of soft continuum manipulators revealed that there is a gap in understanding how to model nonlinearity and hysteresis under dynamic conditions and a lack of material modeling. This article helps to advance the state-of-the-art of dynamic modeling by introducing a novel material-based dynamic model for multichambered soft continuum manipulators that is suitable for real-time control. The model presented is based on the constitutive law of material behavior to capture the true dynamics of the soft actuator. A modified Kelvin–Voigt model is used to embody the visco-hyperelasticity dynamics of soft silicone used in the fabrication of the soft manipulator. The Lagrangian approach is applied to derive the overall equation of motion.

The model has been validated in real-time and the approach presented in this article has accurately captured the nonlinear behavior and hysteresis exhibited by the soft actuator developed in this study. This model can be generalized to other materials by changing the parameters of the constitutive model to fit the stress–strain mechanical test data for any material. The model can also be used for cable-driven soft manipulators by adding additional energy terms representing the action of the cables. Furthermore, a model-based closed loop control was implemented to demonstrate the applicability of the proposed innovative material-based model to feedback control. The designed controller successfully tracked the set trajectory. In future, the model will be optimized and improved to increase the performance of the controller and further investigations will be carried out to apply the theory presented in this article to a multisection soft actuator. Focus will also be given to improve the model specifically for force/interaction control application. The twisting behavior of the soft material needs to be modeled to properly capture the deformation of the material as it interacts with the surrounding. Torsion deformations are one of the challenges addressed by Marchese et al. in extending a single-segment module into a multisegment system.

**Acknowledgments**

The work described in this article was supported by the STIFF-FLOP project grant from the European Commission’s Seventh Framework Program under grant agreement 287728. This project is also partly supported by the Ministry of Education Malaysia, Universiti Kebangsaan Malaysia (UKM).

**Author Disclosure Statement**

No competing financial interests exist.

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