An analytical model for the prediction of load distribution in highly torqued multi-bolt composite joints

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A B S T R A C T

This paper presents the development of an enhanced analytical approach for modelling the load distribution in multi-bolt composite joints. The model is a closed-form extension of a spring-based method, where bolts and laminates are represented by a series of springs and masses. The enhancement accounts for static friction effects between the laminates, a primary mechanism of load transfer in highly torqued bolted joints. The method is validated against detailed three-dimensional finite element models and for static friction effects between the laminates, a primary mechanism of load transfer in highly torqued multi-bolt joints. The model is a closed-form extension of a spring-based method, where possible, experimental results. The effect of varying bolt-torque and bolt-hole clearance on the load distribution in a three-bolt, single-lap joint is investigated and the method proves to be robust, accurate and highly efficient. Finally, the method is employed in a parameter study, where increasing bolt torque levels can be used for achieving a more even load distribution in multi-bolt joints.

1. Introduction

In composite aircraft structures, bolted joints represent areas of localised stress concentration and experience a minimal amount of mechanical softening compared to their metal counterparts. Save small plasticity effects in metallic fasteners, bolted composite joints can fail catastrophically upon excessive loading and tend to limit the structural integrity and load-carrying capacity of aircraft. As a result, the overall approach to design tends to be over conservative, thus causing the composite material to detract from its original weight-saving potential. Inefficient design practices compound this problem, where solutions to bolted joint problems suffer from long lead times and high financial expenditure. In this paper, this problem is addressed through the use of a simple analytical design tool, which can be used for the analysis of load transfer in multi-bolt joints.

In the design of bolted composite joints, many design variables need to be considered. Such variables include fastener position, joint material, fastener type (countersunk or protruding head), hole diameter, joint thickness, bolt-hole clearance and bolt torque. Friction forces exist at each contact interface in joints, however, most of the load is transferred at the shear plane of the joint, and the magnitude of this force is highly dependant on bolt torque or fastener pre-load. Bolt-hole clearance and friction effects often result in an unequal load distribution in the joint, and are thus an important consideration in the design of multi-bolt joints. The main aim of this paper is to produce a generic, yet cost-effective approach for modelling the mechanical behaviour of the joint. Most of the main design variables are considered, with particular focus on varying bolt-hole clearance and friction effects between jointed members.

Current joint design practices involve analytical approaches [1–4], experimental tests [5–7] and three-dimensional finite element (3-D FE) analysis [8–14]. In detailed 3-D FE models, each part of the joint is modelled separately, while contact models with tight contact tolerances are used to model the interaction between each part. Such models capture effects such as clearance, bolt-torque and detailed contact stresses [8–10], friction effects between discrete regions in the joint [11], the load distribution in multi-bolt joints [12] and material damage propagation [13,14]. While such models provide accurate solutions for small joint specimens, they suffer from excessive computation time, especially where friction and damage models are present.

Regarding more efficient design practices, simplified finite element models have been proposed in the past. One such technique was due to Friberg [15], who used linear beam elements to model the fastener and shell elements to model the composite plates. Coupled with a simplified contact model, the method was used to simulate the load distribution in a multi-row, multi-column joint and good agreement was obtained when compared to an experimental test. However, this model omitted important joint parameters, such as friction between the laminates. This issue was later addressed by Ekh and Schön [16], who used beam elements to model both the bolt and the laminates, while bolt-hole clearance and friction effects were captured through the use of connector elements. While excellent agreement was obtained with experimental and numerical results, this approach was limited to
single-column joints. To capture the full mechanical behaviour in three dimensions, Gray and McCarthy [17] used shell elements to model the laminates and a combination of rigid contact surfaces and beam elements to model the bolt. Bolt-hole clearance effects and joint friction were included and again good correlation was obtained with experimental and numerical results for multi-bolt joints.

Alternative, cost-effective techniques have also been proposed for the efficient design of bolted composite joints. These include boundary element formulations [18], boundary collocation methods [19,20] and semi-empirical approaches [21]. Barut and Madenci [22] proposed a semi-analytical model for elastic stress analysis of bolted-bonded, single-lap joints. For the design optimisation of multi-bolt joints, simple spring-based methods [1–4] are a highly efficient, low-cost method of analysis and have thus been investigated for the purposes of this paper. In this paper, the current spring-based method [4] is developed further to account for bolt torque and friction effects between laminates.

2. Problem description

The single-bolt, single-lap joint, shown in Fig. 1a, was used for the initial development of the analytical model as extensive experimental and numerical results for this joint configuration were available from the literature [5,9–11]. In order to assess the model’s accuracy in terms of predicting bolt-load distributions in multi-bolt joints, the three-bolt, single-lap joint shown in Fig. 1b was also examined. Both these joint configurations were used for benchmark studies in the EU Framework 5 project BOJCAS (Bolted Joints in Composite Aircraft Structures) [23] and were designed to fail initially in the bearing mode. The joints were fabricated using a carbon fibre/epoxy composite material manufactured by Hexcel composites, with designation HTA/6376. Both laminates had a quasi-isotropic lay-up with stacking sequence [45/0/-45/90]s. Each ply had a thickness of 0.13 mm, yielding a total laminate thickness of 5.2 mm. The unidirectional lamina material properties are listed in Table 1. However, for this study the material was modelled using homogeneous material properties, which were obtained by McCarthy et al. [9] and are also listed in Table 1. The 8 mm diameter bolts were made from aerospace grade Titanium alloy and the material properties are, again, listed in Table 1.

To illustrate the effect of friction in highly torqued joints, experimental load–displacement curves taken from actual single-bolt, single-lap joint experiments [5] are shown in Fig. 2a. Four different joint configurations are considered, each one accounting for changes in bolt-hole clearance and bolt torque. Firstly, and regarding the 10 μm clearance cases, the highly torqued (16 Nm) joint is seen to be slightly stiffer, but significantly stronger than the finger-tight torqued (0.5 Nm) joint. The reason for this is that the onset of critical bearing damage occurs at higher load levels in the highly torqued joint due to a significant percentage of the joint load being reacted by high frictional forces acting at the shear plane, thus reducing the bearing load acting at the bolt-hole. In the finger-tight torqued joints, these frictional forces are minimal and the bolt reacts essentially all the applied joint load.

Turning to the larger clearance (240 μm) joints, it can be seen that for the finger-tight torqued (0.5 Nm) joint, a delay in load
take-up is clearly visible. It transpires that this delay is approximately equal to the bolt-hole clearance, as would be expected (however, bolt tipping also adds to this displacement). In the highly torqued (16 Nm) joint, this “delay in load take-up” or “clearance” effect occurs at a much higher load level, as can be seen in more detail in Fig. 2b. The initial quasi-linear region, designated \( \text{Slope 1} \), is due to the joint load being reacted solely by static friction forces acting at the shear plane. With increased joint load, these static friction forces are overcome, and the laminates begin to slip relative to each other. During this phase, designated the \( \text{Transition Region} \), the large bolt-hole clearance is taken up and the bolt shank begins to contact the laminates. When significant contact is established between the bolt and the laminates, the bolt starts to transmit load and the stiffness of the joint increases significantly. This region is identified as the \( \text{Slope 2} \) region in Fig. 2b. For all the joints considered in Fig. 2a, these friction and clearance load variations occur early in the loading history and have a significant effect on the bearing strength of the joint (i.e. where the curves deviate from linearity at approximately 16 kN and upwards). In multi-bolt joints, these effects can be even more severe as a delay in load take-up at a particular bolt results in other bolts compensating for it, which in-turn may cause an already highly loaded bolt-hole region to fail catastrophically [6]. This redistribution of joint load is discussed in this paper, where the non-linear elastic behaviour of joints is considered in the load distribution analysis. Joint failure will be considered in a future publication of this method.

### 3. Model development

#### 3.1. The spring-based method

McCarthy et al. [4] idealised bolted joints as a simple spring-mass system, an example of which is illustrated in Fig. 3. The three-bolt, single-lap joint in Fig. 3a may be represented by a system of springs and masses, as shown in Fig. 3b. This approach is applicable to any number of in-line bolts and also double-lap joint configurations. The stiffness of the region designated “Laminate 1” is represented by \( K_{\text{LAM1}} \). When considering double-lap joints, only half the spring stiffness for Laminate 1 and half the total joint load \( P \) are used. It is also assumed by McCarthy et al. [4] that friction effects are negligible. However, the aim of this paper is to extend the current modelling strategy to account for static friction effects.

#### Table 1

<table>
<thead>
<tr>
<th>Material properties for HTA/6376</th>
<th>( E_{11} ) (GPa)</th>
<th>( E_{22} ) (GPa)</th>
<th>( E_{33} ) (GPa)</th>
<th>( G_{12} ) (GPa)</th>
<th>( G_{13} ) (GPa)</th>
<th>( G_{23} ) (GPa)</th>
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</thead>
<tbody>
<tr>
<td>Unidirectional properties</td>
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<td>10</td>
<td>10</td>
<td>5.2</td>
<td>5.2</td>
<td>3.9</td>
</tr>
<tr>
<td>Homogenised laminate properties</td>
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<td>54.25</td>
<td>12.59</td>
<td>20.72</td>
<td>4.55</td>
<td>4.55</td>
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<tr>
<td>Titanium properties</td>
<td>110</td>
<td>0.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Verified by classical laminate theory.

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**Fig. 2.** Experimental load–displacement curve from single-bolt, single-lap joint tests: (a) effect of varying clearance and bolt torque on stiffness and strength; (b) effect of friction on elastic behaviour.
McCarthy et al. [4] examined each mass in the system using free-body diagrams and equations of motion. For example, a free-body diagram for Mass 4 is illustrated in Fig. 4a and its corresponding equation of motion may be written as:

\[
mx_4 - K_{\text{LAM}_1,2}x_2 - K_{\text{BOLT}_2}x_3 + (K_{\text{LAM}_1,2} + K_{\text{BOLT}_2} + K_{\text{LAM}_1,1})x_4 = 0
\]

where \(x_i\) is the x-displacement of Mass \(i\) (\(i = 2, 3, 4, 6\)), \(c_2\) is the bolt-hole clearance at Bolt 2 and \(x_4\) is the acceleration of Mass 4. In this approach only the static response of the joint is considered and so, the nodal accelerations are set to zero (i.e. \(x_4 = 0\) in Eq. (1)). Ignoring dynamic effects and carrying out a similar analysis for the remaining masses yields a system of linear equations of the type:

\[
[K][x] = [F]
\]

where

\[
K = \begin{bmatrix}
K_{\text{LAM}_2,\text{END}} + & -K_{\text{BOLT}_3} & -K_{\text{LAM}_2} & 0 & 0 & 0 & 0 \\
-K_{\text{BOLT}_3} & (K_{\text{BOLT}_2} + K_{\text{LAM}_1,2}) & 0 & -K_{\text{LAM}_1,2} & 0 & 0 & 0 \\
-K_{\text{LAM}_2} & 0 & (K_{\text{LAM}_2,2} + K_{\text{BOLT}_2}) & 0 & -K_{\text{LAM}_2,1} & 0 & 0 \\
0 & -K_{\text{LAM}_1,2} & -K_{\text{BOLT}_2} & (K_{\text{LAM}_1,2} + K_{\text{BOLT}_2}) & 0 & 0 & 0 \\
0 & 0 & -K_{\text{LAM}_2,1} & 0 & K_{\text{LAM}_2,1} & -K_{\text{BOLT}_1} & 0 \\
0 & 0 & 0 & -K_{\text{LAM}_1,1} & -K_{\text{BOLT}_1} & (K_{\text{LAM}_1,1} + K_{\text{BOLT}_1}) & 0 \\
0 & 0 & 0 & 0 & -K_{\text{LAM}_1,1} & -K_{\text{LAM}_1,2} & (K_{\text{LAM}_1,1} + K_{\text{BOLT}_1}) \\
0 & 0 & 0 & 0 & 0 & 0 & K_{\text{LAM}_1,\text{END}}
\end{bmatrix}
\]

\[
[F] = \begin{bmatrix}
-K_{\text{BOLT}_3}c_2 \\
K_{\text{BOLT}_3}c_2 \\
-K_{\text{BOLT}_2}c_2 \\
K_{\text{BOLT}_2}c_2 \\
K_{\text{BOLT}_1}c_1 \\
K_{\text{BOLT}_1}c_1 \\
p
\end{bmatrix}
\]

and

\[
[x] = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7
\end{bmatrix}, \quad [F] = \begin{bmatrix}
-K_{\text{BOLT}_3}c_2 \\
K_{\text{BOLT}_3}c_2 \\
-K_{\text{BOLT}_2}c_2 \\
K_{\text{BOLT}_2}c_2 \\
K_{\text{BOLT}_1}c_1 \\
K_{\text{BOLT}_1}c_1 \\
p
\end{bmatrix}
\]

In double-lap joints, the stiffness of each bolt, defined here as \(K_{\text{BOLT}}\), was derived by Tate and Rosenfeld [1] and Nelson et al. [3]:

\[
\frac{1}{K_{\text{BOLT}}} = \frac{2(t_{\text{LAM}_2} + t_{\text{LAM}_1})}{3E_{\text{BOLT}}A_{\text{BOLT}}} + \frac{2(t_{\text{LAM}_2} + t_{\text{LAM}_1})}{t_{\text{LAM}_2}t_{\text{LAM}_1}E_{\text{BOLT}}} + \frac{1}{t_{\text{LAM}_2}(\sqrt{E_{xx}}E_{yy})_{\text{LAM}_2}} + \frac{1}{t_{\text{LAM}_1}(\sqrt{E_{xx}}E_{yy})_{\text{LAM}_1}}[1 + 3\beta]
\]

where it can be clearly seen that the bending term in Eq. (3) has been removed. Instead, three of the remaining terms are multiplied by a factor \([1 + 3\beta]\), where \(\beta\) term represents the fraction of the bending moment reacted by non-uniform contact stresses in the laminates. In single-lap joints, the remaining fraction of the bending
moment is reacted by the head and the nut and this also induces secondary bending [9,17] in the joint. For example, in joints where no lateral clamping is present, such as pin-loaded joints, all of the bending moment is carried by contact stresses through the thickness of the hole and the value of \( \beta \) would equal 1. The value of \( \beta \) may range from 0.5 for countersunk fasteners to 0.15 for protruding head bolts [4].

The stiffness of the laminates can be quantified using a simpler expression. For example, in \( K_{LAM} \), \( K_{LAM,1} \) may be expressed as:

\[
K_{LAM,1} = \left( \frac{E_{x,y} W t}{p - d} \right)_{LAM,1}
\]

where \( E_{x,y} \) represents the homogenised laminate modulus in the global \( x \)-direction, \( w \) is the laminate width, \( t \) is the laminate thickness, \( p \) is the bolt-pitch and \( d \) is the diameter of the hole.

3.2. Extension of the method to account for bolt torque

Eq. (2) is suitable for joints where lateral clamping effects are low, i.e. finger-tight torque conditions. In this paper, the McCarthy model [4] is enhanced to account for static friction effects between the laminates. The effect of friction at say, Bolt 3, is illustrated in Fig. 2b. The region designated Slope 1, which occurs due to sticking between the laminates, is characterised mainly by the shear stiffness \( (K_{SHEAR}) \) of the laminates. The critical friction load \( (P_{PRIC}) \) is equal to \( K_{SHEAR} \), multiplied by the deflection of the joint due to friction, \( u_{fr} \). For the purposes of the analytical model, it is assumed the length of the transition region is approximately equal to the bolt-hole clearance, \( c_3 \). Detailed 3-D FE models show that changes in contact pressure between the washer and the laminates tend to reduce with increasing joint displacement and play a relatively small role in the overall mechanics of the bolted joint [11]. After the transition region, the curve then follows the path of Slope 2 and the joint load, \( P_2 \), is equal the bolt stiffness \( (K_{BOLT}) \) multiplied by the deflection of the bolt \( (x_2 - x_1 - u_2 - c_2) \) plus the critical friction force \( (P_{PRIC}) \).

In the analytical model, the criterion for sticking-to-slipping, i.e. the change in load–displacement behaviour from Slope 1 to the transition region, is enforced using the classical Coulomb friction law:

\[
|F_1| \leq \mu |F_N|
\]

where the normal and tangential friction forces are referred to as \( F_N \) and \( F_t \), respectively, and \( \mu \) refers to the coefficient of static friction between the laminates. In the context of a bolted joint, \( F_N \) refers to the bolt pre-load. It has been shown in a previous study [24] that instrumented fasteners can be used to quantify values of axial pre-stress in torqued bolts, which in-turn can be used to define \( F_N \). The static friction effects contributing to Slope 1 may be accounted for by replacing bolt stiffness terms, \( K_{BOLT} \), \( K_{BOLT} \) and \( K_{BOLT} \), by laminate shear stiffness terms, \( K_{SHEAR} \), \( K_{SHEAR} \) and \( K_{SHEAR} \), in the stiffness matrix \([K]\) of Eq. (2). The load vector \([\mathbf{F}]\) also changes to

\[
\{\mathbf{F}\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \end{bmatrix}
\]

since static friction forces prevent sliding during this phase of loading.

When the stiffnesses of the bolts have been taken up, the load vector \([\mathbf{F}]\) in Eq. (2) is re-derived to account for static friction forces. For example, the free-body diagram in Fig. 4a is redrawn in Fig. 4b, where the friction forces \( (P_{PRIC}) \) and a delay in load take-up due to friction \( (u_{fr}) \) are accounted for. Eq. (1) may be rewritten as:

\[
\begin{align*}
mx_4 - K_{LAM,2}x_2 - K_{BOLT}x_3 + (K_{LAM,2} + K_{BOLT} + K_{LAM,1})x_4 \\
K_{LAM,1}x_4 + K_{BOLT}x_2 - P_{PRIC}
\end{align*}
\]

This phenomenon changes the load vector to:

\[
\{\mathbf{F}\} = \begin{bmatrix}
-K_{BOLT}c_3 - K_{BOLT}u_3 + P_{PRIC} \\
K_{BOLT}c_3 + K_{BOLT}u_3 - P_{PRIC} \\
-K_{BOLT}c_2 - K_{BOLT}u_2 + P_{PRIC} \\
K_{BOLT}c_2 + K_{BOLT}u_2 - P_{PRIC} \\
K_{BOLT}c_1 - K_{BOLT}u_1 + P_{PRIC} \\
K_{BOLT}c_1 + K_{BOLT}u_1 - P_{PRIC} \\
0 \\
\end{bmatrix}
\]

where \( u_{fr} \) represents the displacement due to friction at each bolt location \( (i = 1-3) \) and \( P_{PRIC} \) represents critical friction force.

The determination of Slope 1 in Fig. 2b relies heavily on an accurate expression for the shear stiffness of the laminates, \( K_{SHEAR} \) \( (i = 1, \ldots, 3) \). In order to predict \( K_{SHEAR} \), the linear elastic strain energy for a volume of material is considered and for transverse shear may be expressed as:

\[
U = \int_V \frac{\tau_{xz}^2 dV}{2G_{xz}} = \frac{\tau_{xz}^2 V}{2G_{xz}}
\]

where \( U \), \( \tau_{xz} \), \( V \) and \( G_{xz} \) refer to energy due to transverse shear, the out-of-plane shear stress in the laminate, the volume of the material and the out-of-plane shear modulus, respectively. Axial loading, bending and twisting have little or no effect on this phase of loading in the joint and are thus omitted from the energy equation. Another assumption is that the contact area is approximately equal to the contact area of a washer or bolt-head \( (A_w) \) as shown in Fig. 5. A simple expression for \( \tau_{xz} \) can thus be used [25]:

\[
\tau_{xz} = \frac{F_t}{A_w}
\]
By considering Eq. (11) and noting that V is simply the planar area of the washer multiplied by the thickness of the laminate, Eq. (10) may be rewritten as:

$$U = \frac{\bar{F}_i t_{LAM}}{2 A_w G_{xz}}$$

where $t_{LAM}$ is the thickness of the laminate. The deflection of the cylindrical portion of the laminate ($\delta$) may be found by differentiating the strain energy with respect to the friction force $\bar{F}_i$, i.e.,

$$\delta = \frac{dU}{d\bar{F}_i} = \frac{\bar{F}_i t_{LAM}}{A_w G_{xz}}$$

The transverse shear stiffness ($K_{SHEAR}$) is simply the friction force ($\bar{F}_i$) divided by the shear deflection, $\delta$. Hence, $K_{SHEAR(i)}$ for each bolt location may be expressed as:

$$K_{SHEAR(i)} = \frac{A_{w(i)} G_{xz(i)}}{t(i)}$$

3.3. Summary of the analytical method

The overall analytical model can be implemented in a computer program through the use three conditional statements:

1. If the critical friction forces, $P_{FRIC(i)}$, have not been exceeded, Eq. (2) is rewritten by replacing the bolt stiffness terms $K_{BOLT(i)}$ by the laminate shear stiffness terms $K_{SHEAR(i)}$ and the displacements ($x_{(j)}$, $j = 1, \ldots, 7$) are resolved by pre-multiplying the load vector in Eq. (7) by the inverse of the stiffness matrix.
2. The transition from sticking-to-slipping is enforced using the classical Coulomb friction model in Eq. (6) and the bolt stiffness terms $K_{BOLT(i)}$ are set equal to zero.
3. If the clearance has been taken up at Bolts (i), the displacements ($x_{(j)}$) are found by pre-multiplying the load vector in Eq. (9) by the inverse of the stiffness matrix in Eq. (2).

On a further note, the load–displacement behaviour of each individual bolt may vary depending on the phase of loading at that bolt. For example, if friction forces are still present at Bolt 1 due to, say, a comparably higher bolt torque than Bolts 2 and 3, Bolt 1 would then satisfy conditional statement 1, while Bolts 2 and 3 would satisfy conditional statement 2. This would result in the presence of both laminate shear stiffness and bolt stiffness terms in the stiffness matrix of Eq. (2) and this greatly improves the flexibility of the analytical model.

4. Results and discussion

In some cases the analytical models were validated using 3-D FE models which were solved using the commercially available finite element code ABAQUS/Implicit [26]. A finite element model of the three-bolt, single-lap joint is illustrated in Fig. 6, where the model includes details such as bolt-heads, nuts and washers. The laminates were modelled using three-dimensional solid elements with improved bending capability (designated C3D8I in ABAQUS [26]). Since a detailed laminate stress analysis of the joint was not of primary interest in this study, the laminates were also modelled using the homogeneous material properties listed in Table 1. For a detailed description of 3-D FE modelling of bolted composite joints involving tight contact tolerances, see McCarthy et al. [9].

All analytical models were created and solved using the MATLAB [27] programming language.

4.1. Model validation

To validate the analytical model, the single-bolt, single-lap benchmark from Fig. 1a was compared to 3-D FE models. This benchmark provides a clear illustration of the mechanical behaviour of a joint in isolation. In order to demonstrate the effect of friction in the joint, both ‘finger-tight’ and ‘torque-tight’ conditions have been compared. For a single-bolt, single-lap joint, the joint stiffness equation (Eq. (2)) may be reduced to:

$$\begin{bmatrix} K_{LAM2} + K_{BOLT} & -K_{BOLT} & 0 \\ -K_{BOLT} & K_{BOLT} + K_{LAM1} & -K_{LAM1} \\ 0 & -K_{LAM1} & K_{LAM1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -K_{BOLT}c \\ K_{BOLT}c \\ -K_{BOLT}c \end{bmatrix}$$

A recommended ‘in-service’ type bolt-torque of 16 Nm or 227 MPa bolt pre-stress [11] was investigated for the torque-tightened condition, while a 0.5 Nm (or 7.2 MPa pre-stress) was applied in the finger-tight models. This pre-stress was implemented using the $F_n$ variable in the analytical model and a bolt pre-tension section [26] in 3-D FE models. The friction coefficient between the laminates was set to 0.42 in all models. This value was obtained from [11].
The resulting load–displacement curves for the finger-tight joints are shown in Fig. 7a and remarkable agreement was obtained between the analytical method and 3-D FE models for the four clearance (denoted by ‘c’) cases considered. The delay in load take-up due to clearance effects was captured using both approaches. The load–displacement curves for the torque-tightened joints are shown in Fig. 7b. There is an initial linear region up to approximately 0.1 mm joint displacement before any clearance in the joint is taken up. As stated in Section 3.2, this linear region (or Slope 1 in Fig. 2b) is dominated mainly by transverse shear deformation in the laminates, which is not present in the finger-tight models in Fig. 7a. In terms of Slope 1, excellent agreement was obtained between the analytical models and the 3-D FE models, providing confidence that the assumptions used in the derivation of $K_{SHEAR}$ in Eq. (14) were correct. The critical friction force ($P_{FRIC}$), between Slope 1 and the transition region, was also predicted quite well by the model and was found to be approximately 5 kN. The length of the transition region was also predicted well by the analytical model, and this is clearly seen in joints where larger bolt-hole clearances were considered (i.e. $c = 160 \mu m$ and $c = 240 \mu m$). As can be seen, the detailed 3-D FE models predicted a slight increase in load during this phase of the loading, which is due to increases in friction between the washers and laminates and small changes in the contact area between the laminates. Such effects were ignored in the analytical model and were considered too difficult to implement for a relatively minor increase in model accuracy. Finally, considering the third linear region (Slope 2) where the stiffness of the bolt is taken up, excellent agreement was obtained between the analytical model and the 3-D FE model for all clearance cases examined.

4.2. Load distribution in a three-bolt, single-lap joint

For further validation of the analytical model, a load distribution study was carried out on the three-bolt, single-lap joint shown in Fig. 1b and the corresponding bolt numbering convention is illustrated in Fig. 6. Initially, the effect of finger-tight torque conditions (0.5 Nm) was investigated for all three bolts, followed by an analysis of torque-tightened bolts (16 Nm). Four different joint configurations were modelled where Bolts 1–3 had different bolt-hole clearances, according to Table 2.

*Table 2*

<table>
<thead>
<tr>
<th>Case</th>
<th>Clearance at Bolt 1 ($\mu m$)</th>
<th>Clearance at Bolt 2 ($\mu m$)</th>
<th>Clearance at Bolt 3 ($\mu m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>160</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
<td>160</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Firstly, the effect of friction on the load–displacement response of the joint was investigated. The results from Case 4 are presented in Fig. 8 and both finger-tight and torque-tight conditions were considered. In the finger-tight case, the load distribution is piece-wise linear, with a change in slope observed at approximately 0.4 mm. The reason why this variation occurs is due to the fact that initially, the lower clearance holes (10 $\mu m$ in both Bolts 2 and 3, ...
respectively) react the entire joint load. At 0.4 mm joint displacement, the clearance at Bolt 1 (the largest of the three clearances at 240 μm) is taken up, resulting in an increase in joint stiffness. In the torque-tight case, the load–displacement curve also varies piece-wise linearly, with a change in slope observed at 0.17 mm and 0.5 mm, respectively. The reason for the first variation is due to the presence of friction between the laminates, which transitions from sticking to slipping at 0.17 mm. The reason for the second variation is due to the larger clearance hole at Bolt 1 being taken up and this bolt reacting some load. Good agreement was obtained between the analytically and numerically predicted load displacement curves for both torque conditions.

For finger-tight torque conditions, the load distribution in the analytical model, shown together with experimental results from instrumented bolts [6], is illustrated for all clearance cases in Fig. 9. For Case 1, the load distribution was captured well with Bolt 1 and Bolt 3 taking a higher portion of the load than Bolt 2 (i.e. the middle bolt). However, the analytical model is slightly stiffer than the experiment and this may be due to the fact that the experiments used corrected machine stroke readings to record the total joint displacement. Concerning Case 2, excellent agreement was obtained between the model and the experiment. In this case, the reaction forces at Bolt 2 are significantly delayed due to its large 160 μm clearance. Turning to Case 3, Bolt 3 is seen to take the majority of the overall joint load due to the large clearances at Bolts 1 and 2. As the displacement of the joint increases, Bolt 1 begins to share some of the load, followed shortly by Bolt 2. Case 4 follows a similar trend to Case 2. In general the analytical model predicted load distribution very well (and certainly well enough for preliminary design investigations) when compared to the experiments of [6].

Turning to torque-tightened joints, the same clearance cases described in Table 2 were examined. Of particular interest here was the analytical model's capability to capture the effect of load transfer in the joint due to static friction forces between the laminates. In this case, the analytical models were compared to 3-D FE models as no experimental results were available and the results are shown in Fig. 10. A number of interesting phenomena occur when friction is present in the joint:

1. Regarding all clearance cases, an initial delay in load take-up by the bolts occurs until approximately 0.2 mm joint displacement, which is due to a linear increase in friction forces in the joint, where sticking occurs between the laminates.
2. Friction results in no relative motion (i.e. sticking) between the laminates up until approximately 16 kN joint load. At this load, the laminates begin to slip relative to each other and the bolts begin to share a percentage of the overall joint load.
3. Friction considerably reduces the magnitude of the bolt-loads at a given joint displacement, which can be clearly seen by

![Fig. 9. Bolt-load distribution for varying clearance cases in finger-tightened three-bolt, single-lap joints – analytical model and experiments [6].](image-url)
comparing Fig. 9 with Fig. 10. For example, at a joint displacement of 0.3 mm in Case 1, the bolt-load at Bolt 1 is 5 kN and 1.5 kN in the finger-tightened joint and the torque-tightened joint, respectively.

The effect of varying clearance was also captured quite well by the analytical model. For example in Case 4, there is a longer delay in load take-up at Bolt 1, due to the presence of the large 240 μm clearance and this can be seen by comparing the analytical and numerical results. In general, good agreement was obtained between the analytical model and the 3-D FE model for all clearance cases considered.

The analytical model was also compared to 3-D FE models in terms of computational efficiency. In order to make a fair comparison, the most basic 3D FE model that could predict load distribution accurately needed to be established and so a number of steps were taken to achieve this. Firstly, writing of all stress, strain and field outputs at all integration points was suppressed to reduce the amount of computing time consumed in writing to journal files. Secondly, a mesh refinement study was carried out in order to determine the minimum mesh density needed to achieve an accurate prediction of load distribution. It was found that very coarse meshes were sufficient to predict the load distribution in the finger-tight torqued joints, where friction forces are minimal, however, very refined meshes are needed to accurately predict the high friction forces at the shear plane in torqued tightened joints. CPU times for the 3-D FE models are listed in Table 3 together with a prediction of the maximum friction force for each clearance case. In Case 1, it can been seen that the two coarse meshes (3715 and 8611 elements, respectively) under-predict the maximum friction force considerably, and this is due to the fact that tangential stresses close to the bolt-hole are not represented well by the model. For the refined mesh (18,115 elements), which is illustrated in Fig. 6, a reasonable approximation of the maximum friction force is obtained and is thus used as a basis for comparison. The reason why such models require a longer is CPU time is due to

<table>
<thead>
<tr>
<th>Clearance case</th>
<th>No. of elements in Mesh</th>
<th>CPU time (s)</th>
<th>Estimation of maximum friction force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3715</td>
<td>134</td>
<td>4.98</td>
</tr>
<tr>
<td>1</td>
<td>8611</td>
<td>882</td>
<td>5.97</td>
</tr>
<tr>
<td>1</td>
<td>18,115</td>
<td>3820</td>
<td>16.2</td>
</tr>
<tr>
<td>2</td>
<td>18,115</td>
<td>5926</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>18,115</td>
<td>5215</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>18,115</td>
<td>6407</td>
<td>–</td>
</tr>
</tbody>
</table>
local force discontinuities at nodal locations, particularly those close to the bolt-holes. The solver thus requires considerable iteration to achieve a converged solution. For all clearance cases, the CPU times range from 3820 to 6407 s. In comparison to the 3-D FE models, the analytical model runs instantaneously, thus highlighting a clear advantage to using this approach, especially for parameter studies.

4.3. Parameter study

In this section, the analytical model is used in a parameter study on the three-bolt, single-lap joint shown in Fig. 1b. The control case used here is Case 1 in Table 2, where all bolts have a nominal bolt-hole clearance of 10 μm. Finger-tight torque conditions (0.5 N m) were considered initially at each bolt, as this condition provides

Table 4
Joint configurations for the three-bolt, single-lap joint parameter study.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Bolt pitch study ( P_{1,2} \text{ (mm)} )</th>
<th>( P_{2,3} \text{ (mm)} )</th>
<th>Hole diameter study ( d_{\text{bolt} 1} \text{ (mm)} )</th>
<th>( d_{\text{bolt} 2} \text{ (mm)} )</th>
<th>( d_{\text{bolt} 3} \text{ (mm)} )</th>
<th>Plate width study ( w \text{ (mm)} )</th>
<th>Bolt torque study ( \text{PS}_{\text{bolt 1}} \text{ (MPa)} )</th>
<th>( \text{PS}_{\text{bolt 2}} \text{ (MPa)} )</th>
<th>( \text{PS}_{\text{bolt 3}} \text{ (MPa)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td>36</td>
<td>8</td>
<td>8</td>
<td>8</td>
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<td>7.2</td>
<td>7.2</td>
<td>7.2</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>48</td>
<td>10</td>
<td>8</td>
<td>10</td>
<td>48</td>
<td>79.6</td>
<td>7.2</td>
<td>79.6</td>
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<tr>
<td>3</td>
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<td>8</td>
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<td>139.3</td>
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<td>8</td>
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<td>188</td>
</tr>
<tr>
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<td>24</td>
<td>8</td>
<td>16</td>
<td>8</td>
<td>120</td>
<td>227</td>
<td>7.2</td>
<td>227</td>
</tr>
</tbody>
</table>

\( \text{a PS – bolt pre-stress.} \)

Fig. 11. Effect of design variables on the load distribution in three-bolt, single-lap joints: (a) bolt-pitch; (b) hole diameter; (c) plate-width; (d) bolt-torque.
a good control case for illustrating the benefits of increasing the torque levels. The aim of this study was to try and balance the load distribution in the joint, since Bolt 2 normally carries a smaller percentage of the overall joint load than Bolts 1 and 3, as shown in Fig. 9 (Case 1). The following design variables are considered:

- Bolt pitch.
- Bolt diameter.
- Laminate width.
- Bolt torque.

The quantities associated with each of these variables are listed in Table 4 and the results from the parameter studies are illustrated in Fig. 11, where the dashed line represents load balance between all three bolts. In each parameter study, Configuration 1 represents the control case. The bolt-load has been expressed as a percentage of the total joint load, and a total joint displacement of 1 mm was applied. The first design variable considered was bolt-pitch. In Table 4, the variables \( p_{1-2} \) and \( p_{2-3} \) denote the bolt-spacing between Bolts 1 and 2 and Bolts 2 and 3, respectively, and the results from this study are shown in Fig. 11a. In Configuration 2, the pitch between each bolt was increased to 48 mm and it can be seen that the load is shared more evenly between each bolt. Decreasing the pitch to 24 mm (Configuration 3) also results in a more even load distribution when compared to the control case. Varying the pitch across the joint, i.e. moving the middle bolt closer to Bolt 3 (Configurations 4 and 5), tends to cause the load distribution to become more unequal, with the most isolated bolt (Bolt 1 in this case) experiencing the highest percentage of the total joint load, as would be expected.

The next design variable considered was bolt diameter and its effect on the load distribution is illustrated in Fig. 11b. Increasing the diameter of the two outer bolts (Configuration 2) tends to have an adverse effect on the load distribution, while increasing the diameter of the centre bolt (Configurations 3 and 4) improves the load distribution substantially. The exception case is Configuration 5, where the centre bolt carries a higher percentage of the load due to the very large bolt diameter at that location. This method of balancing the load distribution appears to be more feasible than the bolt-pitch method, as a larger bolt could be placed at the centre location. However, this would increase the weight of the joint, thus reducing joint efficiency.

Following the bolt diameter study, the effect of plate-width was investigated and the results are shown in Fig. 11c. In general, it is noted that increasing the plate-width improves the balance of the load distribution. This effect is more important in multi-column joints, which are beyond the scope of this paper; however, this issue will be addressed in a future extension of the method.

Finally, turning to the bolt torque study in Fig. 11d, it can be seen that increasing the pre-stress of the two outer bolts tends to balance the load distribution, with Configuration 3 giving the best results. Configurations 4 and 5 result in a higher load acting on Bolt 2, with Bolts 1 and 3 carrying less of the total joint load. This method of balancing the load distribution appears to be the most practical, as geometric quantities in the joint do not have to be varied. However, design for fatigue loading may not allow such unequal pre-stresses to be applied to different fasteners in the joint.

5. Concluding remarks

In this paper, a simple analytical approach for modelling the non-linear elastic behaviour of multi-bolt composite joints has been presented. The model is closed-form and includes most of the main design variables necessary for joint design. Important physical effects such as bolt-hole clearance, bolt-torque and friction have been included in the model. The method has proven to be accurate and robust when compared to numerical and experimental studies on three-bolt, single-lap joints. Results from parameter studies were achieved in seconds, where both geometric and material design variables were modified with ease. It was found that it is possible to equalise the load distribution in a three-bolt, single-lap joint with neat-fit clearances by increasing bolt-pitch, increasing the diameter of the centre bolt or increasing the plate width. However, the simplest method of balancing the load distribution was obtained by increasing bolt torque on the two outer bolts. Changing the bolt-load distribution is, however, not the only mechanism controlling joint failure in multi-bolt joints, as by-pass stresses are also known to play a significant role. However, joints are generally designed to fail initially in bearing, as it is not catastrophic, and so equalising the bearing load at each hole (i.e. the bolt-load at each hole) would appear to be beneficial, especially for limit load design.

Finally, the main limitation of this approach is that multi-column joints cannot be analysed, where any number of bolts may be present in both in-plane directions. However, as part of a larger body of research, this method will be extended to the analysis of multi-row, multi-column joints, where the analytical model will be implemented in a user-defined finite element and this will be reported in a future publication.

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