Characterization of two-parameter fracture properties of portland cement concrete containing reclaimed asphalt pavement aggregates by semicircular bending specimens

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A B S T R A C T

Although few available test results showed that portland cement concrete containing reclaimed asphalt pavement (RAP-PCC) can have equivalent (or even improved) fracture properties and ductility compared to plain PCC, the limitations of the existing experimental methods to test the two-parameter fracture properties (TPFP) of concrete (critical stress intensity factor, $K_{\text{c}}$, and critical crack tip opening displacement, CTOD) have hindered an effective characterization of RAP-PCC's fracture properties and ductility. To provide an easy but effective approach, this paper developed an innovative fracture test using the semicircular bending (SCB) geometry to characterize the TPFP of the studied RAP-PCC mixtures. Based on the results, it is confirmed that addition of RAP improves PCC's CTOD, and $G_{\text{c}}$, despite of a reduction in mechanical strengths. The material length, $Q$, of the RAP-PCC mixture is statistically higher than that of the plain PCC, suggesting that RAP-PCC is a more ductile material. Besides, the theoretical tensile strength, tensile MOE, and bilinear softening curve of the RAP-PCC can be easily obtained using the developed SCB fracture test.

1. Introduction

Potential challenges associated with depletion of good aggregate sources and management of excess reclaimed asphalt pavement (RAP) stockpiles increasingly motivate use of RAP in portland cement concrete (PCC) as a coarse aggregate replacement \cite{1}. The use of RAP in PCC (RAP-PCC) can cause reduction in mechanical strengths \cite{2-8}, which is one of the major reasons for the pessimism about use of RAP-PCC for pavement applications in some existing literature. However, a number of field studies in Iowa, Kansas, Illinois, and some European countries (e.g., France) suggested that portland cement concrete pavement containing RAP aggregates can offer equivalent field performance despite the fact that the strength reduction was invariably observed in the laboratory studies \cite{9-16}. In addition, Brand et al. \cite{17} tested large-scale full-depth as well as two-lift RAP-PCC slabs (1.8 m $\times$ 1.8 m $\times$ 15 cm) and observed that the RAP-PCC slabs exhibited similar to slightly higher flexural load capacity relative to the control slabs. The inconsistency between the measured properties of the RAP-PCC from the lab and the actual performance of the field or large-scale RAP-PCC pavement sections is likely attributed to the size effect \cite{18}. Instead of looking at the material strength alone, the material fracture related properties are probably more relevant for the pavement structure whose size range is within the application of non-linear fracture mechanics. RAP-PCC has been reported to have equivalent (or even improved) toughness, ductility, and fracture properties compared to conventional PCC. Huang et al. \cite{19} demonstrated that the longer and more tortuous crack pattern could help dissipate more energy during the fracture in RAP-PCC. They also found that the RAP-PCC mixtures did not fail as abruptly as the control mixture under the splitting tensile test; the RAP-PCC mixtures were capable to maintain the peak load for a longer time while undergoing a larger deformation. By integrating the area under the splitting tensile load versus deformation curve, Huang et al. \cite{19} found that the toughness of the PCC containing coarse RAP only, fine RAP only, and both coarse and fine RAP was 1.9, 1.3 and 2.3 times of the control mixture after 14 days of moisture curing, respectively. Similarly, Tia et al. \cite{20} conducted the flexural beam test to infer the toughness and ductility of the RAP-PCC mixtures. Although the...
concrete specimens made with RAP failed at a lower stress, the failure strain and the area under the stress versus strain curves increased with the increasing content of RAP. The same findings were also reported by Hassan et al. [21] and Su et al. [22] with the flexural beam test and Superpave indirect tensile tests, respectively. For the RAP-PCC fracture properties, only a limited amount of information is available from the existing publications. The only two available results reported from the literature were both based on the RAP-PCC mixtures using a cement-slag-fly ash ternary blend. Brand et al. [12] tested the fracture properties for the two-parameter fracture model through the single-edge notched beam (SEN(B)) RAP-PCC specimens, while Amirkhanian et al. [23] used the disk-shaped compact tension (DCT) to characterize these fracture properties of the RAP-PCC. Both of the researches concluded that the RAP-PCC had a slightly lower critical stress intensity factor (Kc) but similar critical crack tip opening displacement (CTODc) and fracture energies (Gf and Gt). The reported equivalent (or even improved) properties, such as toughness, ductility, and fracture properties, are believed to be some of the reasons that attributed to the good field/large-scale slab performance of the RAP-PCC pavements.

To facilitate a fracture mechanics-based pavement performance prediction procedure for a better assessment of performance of concrete pavement containing RAP aggregates, effective characterization of the fracture properties of RAP-PCC is considered an indispensable step.

1.1. Two-parameter fracture model

Jenq and Shah [24] proposed a two-parameter fracture model using the elastic response of concrete structures based on the effective elastic crack approach. Their statement states that the crack mouth opening displacement (CMOD) can be divided into an elastic (CMODc) and an inelastic (CMODc) component:

\[ \text{CMOD} = \text{CMOD}_c + \text{CMOD}_i \]  

(1)

The inelastic component can be separated from a loading and unloading procedure of the experiment and will be excluded from the calculation of the fracture properties for the two-parameter fracture model (hereinafter referred to as two-parameter fracture properties (TPFP)). The TPFP are the critical stress intensity factor, Kc, and the critical crack tip opening displacement, CTODc. Both are determined based on linear elastic fracture mechanics (LEFM) theory:

\[ K_c = \frac{1}{2} \sqrt{\pi A_c \beta_c} \left( \frac{A_c}{b} \right) \]  

(2)

\[ \text{CTOD}_c = \text{CMOD}_c \beta_c \left( \frac{A_c}{b} \right) \]  

(3)

Where Ac is the critical effective elastic crack length, which can be solved for from Equation (4) using the elastic component of the CMOD:

\[ \text{CMOD}_c = \frac{4\sigma_c A_c}{E} \left( \frac{A_c}{b} \right) \]  

(4)

And \( \beta_c, \beta, g_i \) are all geometric functions.

From the Jenq and Shah's theory, the TPFP are considered material properties; they are independent from the size and geometries of the structure. At the critical fracture of a material, the following two conditions need to be satisfied. These are the criteria of the two-parameter fracture model:

\[ K_i = K_c \]  

(5)

\[ \text{CTOD} = \text{CTOD}_c \]  

(6)

Where \( K_i \) and CTOD are the stress intensity factor and the crack tip opening displacement of the structure. Both of them are functions of the applied load, structural geometry and size, and crack length.

Using the \( K_c \) and \( \text{CTOD}_c \), some other concrete material properties can be determined. Jenq and Shah [24] introduced a material length parameter, Q, which is expressed as:

\[ Q = \left( \frac{E \text{CTOD}_c}{K_c} \right)^2 \]  

(7)

Q is considered proportional to the size of the fracture process zone for a given material and can be used to quantify the material's brittleness (i.e., a higher Q indicates a more ductile material).

The ranges of the Q were found to be 12.5–50 mm for hardened cement paste, 50–150 mm for mortar, and 150–350 mm for concrete [25].

The theoretical tensile strength, \( f_t \), of a material can be estimated with the following equation [25]:

\[ f_t = 1.4705 \left( \frac{K_c}{Q} \right)^{2} \]  

(8)

The fracture behavior of concrete can be then predicted using finite element methods. The cohesive zone model with a bilinear softening relation is usually used. The bilinear softening curve is shown in Fig. 1.

In the model, two energy parameters, namely the initial fracture energy, \( G_f \), and the total fracture, \( G_T \), are determined as:

\[ G_f = \frac{(K_c^2)^2}{E} \]  

(9)

\[ G_T = \int_{0}^{t_f} \frac{P d\delta}{t U} \]  

(10)

Where \( \delta_f \) is the ultimate load point displacement when the load \( P = 0 \), \( t \) is the specimen thickness and \( U \) is the length of the uncracked ligament.

The location of the kink point in Fig. 1 is determined as [26]:

\[ \psi = 1 - \frac{\text{CTOD}_c \delta}{2G_T} \]  

(12)

The \( \psi \) in Fig. 1 is the crack opening displacement at the peak load, while \( \psi_i \) is the ultimate opening displacement. The displacement parameters are:

\[ w_k = \frac{2G_k}{f_t} \]  

(13)

\[ w_i = \frac{2}{\psi_i} \left[ G_T - (1 - \psi)G_f \right] \]  

(14)
1.2. Review of available TPFP test methods

1.2.1. RILEM method using a three-point bend beam specimen

The International Union of Laboratories and Experts in Construction Materials, System and Structures (RILEM) Technical Committee 89-FMT on Fracture Mechanics of Concrete-Test Method recommended to test concrete TPFP through a single-edge notched beam (Fig. 2). The recommendations on the dimensions of the concrete beam sample include (1) a span to depth ratio \( \frac{S}{b} \) of 4, (2) the initial notch-to-depth ratio of \( \frac{1}{3} \) and (3) the width of notch is less than 5 mm. The test requires a minimum of 4 specimens for each type of material.

A closed-loop testing machine using the CMOD as the control signal or a relatively stiff machine is required to ensure a stable failure of the sample. A clip-on gage or a linear variable differential transformer is used to record CMOD. The rate of the loading shall be controlled so that the peak load can be reached in about 5 min. The test follows the following steps [25]:

1. Load the specimen monotonically up to the maximum load
2. Manually release the load when it passes the maximum load and within 95% of the maximum load
3. When the applied load is reduced to zero, reload the specimen until sufficient data is recorded

Fig. 3 shows a typical load versus CMOD curve with the loading and unloading cycle. The modulus of elasticity (MOE), \( E \), calculated from the loading portion of the curve is written [12]:

\[
E = \frac{6 Sa g_i (a_0)}{C_i b^2 t} \tag{15}
\]

Where \( C_i \) is the initial compliance calculated from the load versus CMOD curve

\( t \) is the beam width

\( b \) is the beam height

\( S \) is the span length

\( g_i(a) \) is the geometric function of the crack length ratio, \( a = \frac{A}{b} \), \( A \) is the notch length.

\[
g_i(a) = 0.76 - 2.28 a + 3.87a^2 - 2.04a^3 + \frac{0.66}{(1 - a)^2} \tag{16}
\]

From the unloading portion of the curve:

\[
E = \frac{6 Sa g_i (a_c)}{C_c b^2 t} \tag{17}
\]

By equating Equations (15) and (17), the value of the effective-elastic critical crack length ratio, \( a_c \), can be solved for with the following expression:

\[
a_c = \frac{a_0 C_i g_i (a_0)}{C_i g_i (a_c)} \tag{18}
\]

The critical stress intensity factor is then calculated based on the LEFM theory:

\[
K_{lc}^2 = 3(P_c + 0.5W_h) \frac{S \sqrt{\pi a_c g_i (a_c)}}{2b^2 t} \tag{19}
\]

Where \( P_c \) is the peak load and

\[
W_h = \frac{W_{ho} S}{L} \tag{20}
\]

Where \( W_{ho} \) is the self-weight of the beam.

\( L \) is the beam length

The geometry function, \( g_i \), is written as:

\[
g_i(a) = \frac{1.99 - (a_c)(1 - a)(2.15 - 3.93a_0 + 2.70(a_0)^2)}{\sqrt{\pi (1 + 2a_0)(1 - a)^{3/2}}} \tag{21}
\]

The critical crack tip opening, CTOD\(_c\), is then calculated using Equation (22):
2. Research significance

While testing TPFP of concrete can be possible using a SEN(B) or a DCT specimen, neither of the tests is easy to perform: the SEN(B) uses large beam samples, which requires a considerable amount of materials and labor. Additionally, beam shaped specimens are hard to obtain from the field. For the DCT sample, the complicated geometry causes tediousness and requires high accuracy in sample preparation. Accordingly, a simple but effective specimen geometry is heavily needed. Semicircular bending (SCB) specimen has been widely used to characterize fracture properties of asphalt concrete [27–30] and other construction materials [31–33]. The geometry is simple to fabricate from both lab-made cylindrical specimens and field cores; fracture tests for different fracture modes can be easily designed by changing the support locations and crack angles of the specimens as well [34]. In this study, an innovative method using specimens with SCB geometry to test the TPFP has been developed. With the developed SCB fracture test, the TPFP of the RAP-PCC made with two different coarse RAP at the replacement level of 40% were characterized. Some other important parameters were also calculated based on the TPFP. The developed SCB fracture test can serve as an easy and effective method for characterizing concrete’s fracture properties. By comparing the SCB test results with the SEN(B) test results, the newly developed SCB fracture test approach was validated. The results presented in this study provided valuable evidence of the improvement of PCC’s fracture properties with addition of RAP.

3. Materials and mix design

A commercially available Type I/II cement made by TXI Cement Co. and a class F fly ash collected from a fly ash limestone plant in Jewett, Texas were used as cementitious materials. A typical mid-range water reducer and an air entraining agent were selected as chemical admixtures. The virgin coarse aggregate (CA) was limestone with #4 (#57 in ASTM C33 [35]) gradation specified in the TxDOT standard specifications [36]. The fine aggregate (FA) was a concrete natural siliceous sand with satisfied gradation requirements by the standard specification. Two types of RAP were collected from two different Texas districts. The RAP from the Houston district (labeled as HOU) is a coarse RAP containing high amounts of coarser size fraction; it has the similar gradation requirements by the standard specifications [36]. The geometry is simple to fabricate and unloading and loading procedure specified in the RILEM procedure. The modulus of elasticity of concrete is obtained in Equation (23):

\[ E = \frac{2V_{CMOD}(a_c)}{C_{u}a_t} \]  

Where

\[ V_{CMOD}(a_c) = \frac{501.8a^3 + 2294a^2 + 4393 a + 1384}{a^3 + 272.2a^2 - 139.8a^2 - 569.3 a + 433.9} \]  

Similarly, the critical effective-elastic crack length ratio \( a_c \) can be solved for by assuming the equivalency of loading and unloading modulus of elasticity:

\[ V_{CMOD}(a_c) = \frac{C_{u}V_{CMOD}(a_c)}{C_{i}} \]  

The critical stress intensity factor and the critical crack tip opening displacement are calculated as:

\[ K_{iC} = \frac{P}{Wt} {\sqrt{W}} F(a_c) \]  

\[ \text{Where} \]

\[ W \text{ is the length of the DCT specimen} \]

\[ F(a) = -1.498a^3 + 4.569a^2 - 1.078 a + 0.113 \]

\[ a^3 - 2.408a^3 + 1.717a^2 - 0.3467 a + 0.0348 \]  

\[ \text{CTOD}_{C} = \frac{2PV_{CTOD}(a_c)}{E} \]  

Where

\[ V_{CTOD}(a_c) = \frac{6.639a^3 - 3.209a^2 + 0.4169 a - 0.006899}{a^3 - 2.429a^3 + 1.897a^2 - 0.5137 a + 0.04504} \]  

1.2.2. Amirkhanian et al. method using a disk-shaped compact tension specimen

Analogous to the RILEM method, Amirkhanian et al. [23] designed a test to characterize the TPFP of concrete using a disk-shaped compact tension geometry to overcome the difficulty of extracting beams from the field. The test set-up is shown in Fig. 4. This method uses a same loading and unloading procedure specified in the RILEM procedure. The modulus of elasticity of concrete is obtained in Equation (23):

\[ E = \frac{2V_{CMOD}(a_c)}{C_{u}a_t} \]  

Where

\[ V_{CMOD}(a_c) = \frac{501.8a^3 + 2294a^2 + 4393 a + 1384}{a^3 + 272.2a^2 - 139.8a^2 - 569.3 a + 433.9} \]  

Similarly, the critical effective-elastic crack length ratio \( a_c \) can be solved for by assuming the equivalency of loading and unloading modulus of elasticity:

\[ V_{CMOD}(a_c) = \frac{C_{u}V_{CMOD}(a_c)}{C_{i}} \]  

The critical stress intensity factor and the critical crack tip opening displacement are calculated as:

\[ K_{iC} = \frac{P}{Wt} {\sqrt{W}} F(a_c) \]  

\[ \text{Where} \]

\[ W \text{ is the length of the DCT specimen} \]

\[ F(a) = -1.498a^3 + 4.569a^2 - 1.078 a + 0.113 \]

\[ a^3 - 2.408a^3 + 1.717a^2 - 0.3467 a + 0.0348 \]  

\[ \text{CTOD}_{C} = \frac{2PV_{CTOD}(a_c)}{E} \]  

Where

\[ V_{CTOD}(a_c) = \frac{6.639a^3 - 3.209a^2 + 0.4169 a - 0.006899}{a^3 - 2.429a^3 + 1.897a^2 - 0.5137 a + 0.04504} \]  

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Fig. 4. DCT fracture test set-up (Reprinted from Ref. [23]).

Fig. 5. Gradation of the aggregate materials.
Table 1
Aggregate materials characterization.

<table>
<thead>
<tr>
<th>RAP/Aggregate ID</th>
<th>Stone type</th>
<th>Asphalt content (%)</th>
<th>Dry unit weight (kg/m³)</th>
<th>Oven dry specific gravity</th>
<th>Absorption (%)</th>
<th>Abrasion loss by micro-deval (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>Limestone with minor chert particles</td>
<td>na</td>
<td>1551</td>
<td>2.51</td>
<td>2.79</td>
<td>22.84</td>
</tr>
<tr>
<td>FA</td>
<td>Siliceous river sand</td>
<td>na</td>
<td>–</td>
<td>2.58</td>
<td>2.06</td>
<td>na</td>
</tr>
<tr>
<td>HOU</td>
<td>Gravel made of mostly limestone with some siliceous particles</td>
<td>4.00</td>
<td>1335</td>
<td>2.41</td>
<td>2.61</td>
<td>26.54</td>
</tr>
<tr>
<td>BRY</td>
<td>Limestone with few siliceous particles</td>
<td>6.19</td>
<td>1373</td>
<td>2.36</td>
<td>1.78</td>
<td>17.54</td>
</tr>
</tbody>
</table>

The PCC mixtures were designed according to the TxDOT standard specifications for producing a typical Class P concrete [36]. A 0.40 w/cm ratio and a 309 kg/m³ cementitious content (20% fly ash replacement) were adopted for all the mixtures produced in this study. The amount of air entraining agent was selected to create an air content of 5.0% in the mixtures. The coarse RAP replaced the virgin aggregate at 40% replacement level by volume. The mix design is shown in Table 2. The mix ID in this study was assigned with the following format:

w/cm_cementitious content*_replacement level + RAP type

*Since all the mixes were designed according to the TxDOT specifications, which use the U.S customary units, the cementitious content in the mix ID was assigned as 520 (lb/cy, equal to 309 kg/m³).

As an example: 0.40_520_40HOU represents a mix has 0.40 w/cm ratio, 520 lb/cy cementitious content and with HOU RAP replacing 40% of virgin coarse aggregate. Specially, 0.40_520_REF represents the plain concrete mix containing 100% virgin aggregate that has 0.40 w/cm ratio and 520 lb/cy cementitious content.

4. Fracture test

4.1. Specimen preparation

The RAP-PCC mixtures and the control mixture were produced according to the normal practice of making conventional concrete in the lab. Cylindrical specimens of 150 mm in diameter and 300 mm in length were cast and then cured in a standard moisture room with a temperature of 23 °C and a relative humidity (RH) of 100% until 27 days after being cast. The cylindrical specimens were then cut into two halves along the diameter. For each half piece, 8 SCB specimens with a thickness of 38 ± 3 mm were produced; the specimens made from both top and bottom portions of the cylinder were discarded to avoid potential segregation problems. Accordingly, 12 SCB specimens were adopted for each cylindrical specimen (Fig. 6). It is worth noting that the assumption in the calculation of the TPFP is that the specimen exhibits plane stress condition in the test, which requires a relatively thin specimen. The 38-mm specimen thickness was selected by considering both the plane stress assumption and the maximum aggregate size used in the studied mixtures (25 mm). After the SCB specimens were produced, a 3.0 mm wide notch was made in the middle of each SCB sample using a table saw. Two notch lengths were chosen to evaluate the effect of the notch length in this study: one was 38 mm according to a previous study [38]; the other was 12 mm by referring to the AASHTO TP 105 standard method [37], which is used to determine the fracture energy of asphalt mixtures using the SCB geometry. The corresponding notch to radius ratio (A/R) for these two types of specimens were 0.5 and 0.16, respectively. The allowable deviation limits for the notch length are specified as 0.4 mm for the 12 mm and 1.3 mm for the 38 mm. Two knife edges were attached on the notched SCB specimen using a superglue after the specimen was air dried for one day. The knife edges would be used for mounting a clip-on gauge during the test. Pictures of the notched SCB specimens are shown in Fig. 7.

4.2. Testing procedures

A stiff MTS machine with sufficient resolution was used to carry out the test. The CMOD was recorded by the clip-on gauge. In the study, the bending span to specimen radius ratio (S/R) was selected as 1.6, which is the most commonly used value from the previous literature [38]. A picture of the SCB test set-up is shown in Fig. 8.

The loading of the SCB specimens followed the same procedures in the RILEM method described in Section 1.2.1. Because the frame of the MTS is very stiff, the test was controlled with a constant movement of the crosshead at 0.05 mm/min instead of setting up a constant clip-on gauge extension rate. The testing procedure included a loading and an unloading step in order to separate the elastic and inelastic portion of the measured CMOD: the SCB specimen was first loaded monotonically up to the peak load at a constant movement of the crosshead at 0.05 mm/min; as soon as the load reached the peak, an unloading step was applied at the same crosshead movement rate (0.05 mm/min). When the applied load reduced to zero, the specimen was reloaded until sufficient data was recorded.

4.3. Testing results

For each mixture type, 6 SCB specimens with 12 mm notch length (A/R = 0.16) and 6 SCB specimens with 38 mm notch length (A/R = 0.5) were initially prepared. However, a few test results were discarded due to some unexpected errors occurred during either the sample preparation or the test performing stage; but still, at least four specimens were successfully tested for each combination of mixture type and notch length. Typical load versus CMOD curves for the 12 mm notched specimen and the 38 mm notched specimen are shown in Fig. 9. Most of the specimens with 12 mm notch suffered from a sudden failure after the peak load, as such applying an unloading process when the load was still within 95% of the maximum load was not possible. As a result, the unloading process usually occurred when the load already dropped to less than half of the peak load (Fig. 9 (a)). This violation of the test requirement that starting the unloading process when the load was still near the peak caused a higher unloading compliance, consequently resulting in a higher calculated effective elastic crack length.

Table 2
Mix design.

<table>
<thead>
<tr>
<th>Materials</th>
<th>0.40_520_REF</th>
<th>0.40_520_40HOU</th>
<th>0.40_520_40HOU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement (kg/m³)</td>
<td>247</td>
<td>247</td>
<td>247</td>
</tr>
<tr>
<td>Fly ash (kg/m³)</td>
<td>62</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>Virgin coarse aggregate (kg/m³)</td>
<td>1058</td>
<td>604</td>
<td>611</td>
</tr>
<tr>
<td>RAP (kg/m³)</td>
<td>0</td>
<td>403</td>
<td>408</td>
</tr>
<tr>
<td>Fine aggregate (kg/m³)</td>
<td>769</td>
<td>804</td>
<td>783</td>
</tr>
<tr>
<td>Water reducer (ml/m³)</td>
<td>402</td>
<td>402</td>
<td>402</td>
</tr>
<tr>
<td>Air entraining agent</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>(ml/m³)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water (kg/m³)</td>
<td>123</td>
<td>123</td>
<td>123</td>
</tr>
</tbody>
</table>
Therefore, the calculated values of $K_{I_c}$, and CTOD$_c$ were higher than what they should be. Accordingly, the calculated values for $K_{I_c}$, CTOD$_c$, and $G_F$ for all specimens with the 12 mm notch length were considered invalid. Besides, during the unstable failure of the specimens with 12 mm notch, a considerable amount of energy was converted to sound and heat energy, so the area under the load versus load point displacement curve might overestimate the total fracture energy ($G_F$) of the specimen. With the aforementioned reasons, the calculated values for $K_{I_c}$, CTOD$_c$, $G_I$ and $G_F$ for the specimens with 12 mm notch length were not included in the test results; only the values for the specimens with 38 mm notch length were adopted because the specimens exhibited stable failures (Fig. 9 (b)). While a notch length which is too short is not preferable for this test (not only because of the sudden failure at the peak load, but it may also result crack propagation to deviate highly from the centerline of the specimen to be considered mode I fracture), a notch length which is shorter than 38 mm (e.g., 25 mm) can offer a bigger uncracked ligament and hence yield results with lower coefficient of variation (CV). Investigation of the optimum notch length for this SCB test is considered an important aspect for the future work.

The values of the peak load and the CMOD at the peak load for the SCB specimens with 12 mm notch length are still valid because they were obtained prior to the reloading process. The peak load and the CMOD at the peak load for the SCB specimens with two notch lengths are compared in Fig. 10 and Fig. 11, respectively. The peak load of the plain PCC was higher than that for the RAP-PCC mixtures for both RAP types. By comparing the results between the 0.40_520_40HOU mixture and the 0.40_520_40BRY mixture, the PCC mixture containing HOU RAP showed higher load capacity. In the case of the CMOD at the peak load, all the RAP-PCC mixtures exhibited higher values than the plain PCC, indicating that RAP-PCC is more ductile.

5. Analysis and discussion

5.1. Fracture properties

The fracture properties of the studied mixtures including $K_{I_c}$, CTOD$_c$, $G_I$ and $G_F$ are calculated according to the approach explained in Appendix A. As has been mentioned, the results are reported only based on the data for the specimens with 38 mm notch because most of the specimens with 12 mm notch had unstable failures. Brand et al. [12] and Amirkhanian et al. [23] obtained the fracture properties of the ternary blended RAP-PCC mixtures using the SEN(B) and DCT specimens, respectively. A summary of the test results from the current study...
in comparison with those from Brand et al. [12] and Amirkhanian et al. [23] is presented in Table 3.

From Table 3, although the SEN(B) and DCT test results showed that the RAP-PCC samples had slightly lower $K_{IC}$, the current SCB test results indicated that the $K_{IC}$ values were approximately same between the control mixture and the two RAP-PCC mixtures. The $K_{IC}$ tested in this current study was significantly smaller than those using the other two testing methods, but the values were still within the range of typical stress intensity factor for the PCC material [39]. The difference for the different $K_{IC}$ measurements in Table 3 is likely due to the specimens' age and mix design (The other two testing samples were both ternary blends, while the current study used a conventional mix design with fly ash), as well as the intrinsic difference associated with the specimen geometry and size. For the critical crack tip opening displacement, $C_{TOD}$, almost all the tested RAP-PCC samples had higher values than their control samples, indicating that RAP-PCC is a more ductile material. By comparing the SCB results with the existing results from the literature, it is recommended to set the coefficient of variation limits as 20% for the $K_{IC}$ and 40% for the $C_{TOD}$ for a valid concrete fracture test. Regarding the fracture energies, the RAP-PCC generally had comparable or higher initial fracture energy ($G_f$) and total fracture energy ($G_F$) from the literature findings. The test results from this study invariably showed an increase in both initial and total fracture energy in the RAP-PCC relative to the control PCC. It is noted here that the BRY PCC sample showed better fracture properties in terms of all tested parameters relative to the HOU PCC sample. The BRY RAP has a higher asphalt content than the HOU RAP (Table 1), so the 0.40_520_40BRY has a higher total asphalt content than the 0.40_520_40HOU. Based on the results, it appears that the higher the asphalt content, the more ductile the material. According to the authors' previous publications [7,8], the 0.40_520_40BRY is qualified as a dense-graded PCC mixture, while the 0.40_520_40HOU is gap-graded; the better fracture properties of the PCC containing BRY RAP might be partially attributed to the dense aggregate gradation as well.

Table 3
Summary of the fracture properties from the current study in comparison with those from the available literature.

<table>
<thead>
<tr>
<th>Study</th>
<th>Test</th>
<th>RAP replacement level (%)</th>
<th>Age (days)</th>
<th>$K_{IC}$ (MPa·m$^{1/2}$)</th>
<th>$C_{TOD}$ (mm)</th>
<th>$G_f$ (N/m)</th>
<th>$G_F$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand et al.</td>
<td>SEN (B)</td>
<td>0</td>
<td>100</td>
<td>1.267 (5%)</td>
<td>0.016 (10%)</td>
<td>44.7 (12%)</td>
<td>100.4 (15%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>100</td>
<td>1.140 (3%)</td>
<td>0.016 (5%)</td>
<td>43.7 (13%)</td>
<td>86.4 (15%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>35</td>
<td>100</td>
<td>0.974 (8%)</td>
<td>0.014 (36%)</td>
<td>35.8 (21%)</td>
<td>106.5 (15%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>100</td>
<td>1.054 (9%)</td>
<td>0.019 (21%)</td>
<td>47.7 (11%)</td>
<td>113.5 (14%)</td>
</tr>
<tr>
<td>Amirkhanian et al.</td>
<td>DCT</td>
<td>0</td>
<td>142</td>
<td>1.33 (8%)</td>
<td>0.0167 (8%)</td>
<td>49.1 (15%)</td>
<td>120.3 (30%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21</td>
<td>142</td>
<td>1.14 (10%)</td>
<td>0.0176 (15%)</td>
<td>42.2 (18%)</td>
<td>119.0 (17%)</td>
</tr>
<tr>
<td>Current</td>
<td>SCB</td>
<td>0</td>
<td>28</td>
<td>0.615 (12%)</td>
<td>0.0097 (20%)</td>
<td>15.2 (28%)</td>
<td>84.2 (14%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40 (HOU)</td>
<td>28</td>
<td>0.604 (11%)</td>
<td>0.0181 (30%)</td>
<td>25.9 (17%)</td>
<td>91.9 (18%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40 (BRY)</td>
<td>28</td>
<td>0.674 (9%)</td>
<td>0.0244 (21%)</td>
<td>30.4 (13%)</td>
<td>108.9 (24%)</td>
</tr>
</tbody>
</table>

Note: Mean of each test result followed by its coefficient of variation in the parentheses is presented in the table.
To eliminate the bias from the simple comparison among the averaged values, an evaluation of data through statistical approaches was performed for the fracture properties tested in this study. The two-sample t-test (by assuming equal variance) was carried out to assess the statistical significance between different comparisons of each two mixture types for the calculated $K_{IC}$, CTOD, $G_{F}$ and $G_{T}$, respectively. The p-values at a 95% confidence level, for the null hypothesis that the calculated property between the compared two mixtures is equal, are presented in Table 4. From Table 4, the p-value for the $K_{IC}$ of any two-sample comparison is larger than 0.05, which means that there is no statistical evidence to reject the null hypothesis that the $K_{IC}$ values are equal. Therefore, the $K_{IC}$ for different mixtures tested in this study are statistically similar. This result matches the previous conclusion by comparing the averaged values. For the CTOD, p-values less than 0.05 are obtained for all the two-sample comparisons, indicating that there is a 95% confidence to reject the null hypothesis that the compared CTOD values are equal. So, the tested values are indeed significantly different between each two-sample comparison. It is noted that the p values of the CTOD, for the 0.40_520_REF versus the 0.40_520_40HOU and the 0.40_520_REF versus the 0.40_520_40BRY are extremely low. A lower p-value suggests that the difference is more pronounced, so the CTOD of the RAP-PCC mixture (either with HOU RAP or BRY RAP) and that of the plain PCC are much more different, when compared to the difference between the RAP-PCC containing BRY RAP and the RAP-PCC containing HOU RAP. For the $G_{F}$, the difference between the RAP-PCC mixtures and the plain mixture is significant too, but a same conclusion could not be drawn between the two RAP-PCC mixtures. It is interesting to see the p-values for the $G_{F}$ comparisons are all above 0.05, indicating the calculated $G_{F}$ of the studied mixtures do not have differences which are statistically significant enough.

### 5.2. Theoretical tensile strength

Concrete theoretical tensile strength, $f_t$, is the only strength parameter required in the bilinear cohesive zone model. The $f_t$ is considered as a material property which is independent of the size and structure of the specimen. There are two types of approaches to estimate the $f_t$ of PCC. One approach is the direct tension test, whose application is impeded by the challenges and difficulties of grasping concrete specimen due to the brittle nature of concrete material. Instead, alternative test methods such as splitting tensile test (ASTM C496 [40]) and flexural bending test (ASTM C78 [41] and ASTM C293 [42]) are more commonly used as indirect measurements of concrete tensile properties, but both of them tend to overestimate the $f_t$ of PCC [43]. In addition, the specimen size effect might also lead to significant derivation of the test value from the theoretical value for concrete tensile strength. On the other hand, the $f_t$ might be theoretically determined from the failure stress of an infinitely-large uniaxial tensile plate with double-edge crack. However, it is not possible to make an infinitely-large plate in the lab, so the actual tested value ($f_t$) from a finite plate is always larger than the $f_t$.

Although challenges exist in getting close approximation of the $f_t$ of PCC from either direct or indirect tension tests, the $f_t$ might be theoretically determined by substituting the values of $K_{IC}$ and CTOD into the expressions for the stress intensity factor and the crack opening displacement for an infinitely-large uniaxial tensile plate with double-edge crack. The expression has been shown in Equation (8). Using Equation (8), the $f_t$ of the 0.40_520_REF, the 0.40_520_40HOU and the 0.40_520_40BRY are calculated and shown in Table 5. The $f_t$ values are compared with the values from both the flexural test (MOR) and the splitting tension test (STS) conducted in the authors’ previous publications [7,8]. Table 5 shows that the determined $f_t$ values from the fracture test were invariably lower that the measured values from either flexural test or splitting tension test. This finding verified the statement that the measured PCC tensile strength, $f_t$, from indirect approaches overestimates the $f_t$ of PCC. For the conventional PCC, the directly measured tensile strength $f_t$ is commonly accepted to be about 75% of the MOR or 85% of the STS. From Table 5, both MOR and STS are about twice of the $f_t$ determined from the fracture test. The inconsistency might be explained as that the $f_t$ in the previously reported relations (i.e., 75% of the MOR or 85% of the STS) might be obtained from a direct tension test using lab sized specimens (i.e., not infinitely large), which naturally overestimates the theoretical tensile strength $f_t$ due to the specimen size effect.

### 5.3. Material length

The material length, Q, for the studied mixtures is calculated according to Equation (7) and is presented in Fig. 12. It is clear that the RAP-PCC mixtures have a higher material length than that of the plain PCC. Despite of suffering from a high variance, the averaged Q value of the 0.40_520_40BRY is almost twice of that of the 0.40_520_REF. A two-sample t-test was conducted and the result shows that the material length of the 0.40_520_40BRY is significantly higher than that of the 0.40_520_40HOU and the 0.40_520_REF (Table 6). Since Q is an index for material brittleness and the higher the Q the more ductile the material, the improvement of PCC ductility with the addition of RAP has been proved. According to Shah et al. [25], the Q value for concrete ranges from 150 to 350 mm; the test results showed a very good agreement with this range, and this again validated the test results from this new SCB fracture test.

#### Table 4
Two-sample t-test results for the calculated fracture properties (p-value).

<table>
<thead>
<tr>
<th>Mixture comparison</th>
<th>$K_{IC}$</th>
<th>CTOD</th>
<th>$G_{F}$</th>
<th>$G_{T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40_520_REF VS 0.40_520_40HOU</td>
<td>0.8294</td>
<td>0.0112</td>
<td>0.0075</td>
<td>0.4634</td>
</tr>
<tr>
<td>0.40_520_40HOU VS 0.40_520_40BRY</td>
<td>0.8227</td>
<td>0.0111</td>
<td>0.0088</td>
<td>0.1214</td>
</tr>
<tr>
<td>0.40_520_40HOU VS 0.40_520_40BRY</td>
<td>0.1226</td>
<td>0.0449</td>
<td>0.1250</td>
<td>0.2497</td>
</tr>
</tbody>
</table>

#### Table 5
Comparison of $f_t$, MOR and STS of the studied mixtures.

<table>
<thead>
<tr>
<th>Mix</th>
<th>$f_t$ (MPa)</th>
<th>CV</th>
<th>MOR (MPa)</th>
<th>CV</th>
<th>STS (MPa)</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40_520_REF</td>
<td>2.28</td>
<td>9%</td>
<td>4.46</td>
<td>1.95</td>
<td>6%</td>
<td>4.39</td>
</tr>
<tr>
<td>0.40_520_40HOU</td>
<td>2.10</td>
<td>11%</td>
<td>3.39</td>
<td>1.61</td>
<td>4%</td>
<td>3.79</td>
</tr>
<tr>
<td>0.40_520_40BRY</td>
<td>1.86</td>
<td>13%</td>
<td>3.67</td>
<td>1.97</td>
<td>7%</td>
<td>3.95</td>
</tr>
</tbody>
</table>

Note: The (MOR or STS)/$f_t$ ratio is included in the parentheses.
Table 6
Two-sample t-test results for the calculated material length (p-value).

<table>
<thead>
<tr>
<th>Mixture comparison</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40,520,REF VS 0.40,520,40HOU</td>
<td>0.0702</td>
</tr>
<tr>
<td>0.40,520,REF VS 0.40,520,40BRY</td>
<td>0.0321</td>
</tr>
<tr>
<td>0.40,520,40HOU VS 0.40,520,40BRY</td>
<td>0.0360</td>
</tr>
</tbody>
</table>

5.4. Modulus of elasticity

The modulus of elasticity of the studied mixtures can be calculated via Equation (A.32) in Appendix A. Fig. 13 compares the MOE calculated from the SCB fracture test using specimens with both 38 mm notch and 12 mm notch and the MOE tested from the compression test according to ASTM C469 [44] (data can be found in Refs. [7,8]). It is interesting to see that the calculated MOE from the SCB fracture test is dependent on the specimen notch size. The samples with the smaller notch length have higher calculated MOE values. The MOE estimation from the compression test is close to the calculated value using the SCB specimens with 12 mm notch. If the commonly accepted assumption that the tensile MOE and the compressive MOE are same for concrete material is considered, the SCB specimen with a notch length of 12 mm can provide a better estimation of MOE than that with a notch length of 38 mm. However, this assumption of the equivalence of tensile MOE and compressive MOE is not necessarily valid for the tangent modulus used in this study since the contact between matrix largely determines tensile properties of PCC while the interaction of aggregate and matrix controls PCCs behavior under compression. As matrix is usually less stiff than aggregate, tensile MOE of PCC should be smaller than compressive MOE of PCC. Fig. 13 confirms that tensile MOE is lower than compressive MOE for PCC materials [45]. It is also noted that the use of Equation (A.32) to calculate E assumes that the specimen exhibits plane stress behavior. If the specimen exhibits plane strain behavior, the calculated MOE value equals to \( \frac{E}{1+\nu} \). For the studied SCB geometries, the actual stress field in the specimen should be neither a plane stress nor a plane strain, so the estimated value of MOE is within a range of E and \( \frac{E}{1+\nu} \). It is reasonable to consider that the determined MOE using the current method is close to E because the difference between E and \( \frac{E}{1+\nu} \) is less than 3% for a typically concrete material with a Poisson's ratio of 0.15.

5.5. Parameters for the cohesive zone model

The essential fracture parameters in the bilinear softening curve are calculated according to Equation (11)-(14). The results are shown in Table 7. Both of two RAP-PCC mixtures are reported to have higher displacement parameters (i.e., \( w_1, w_2, \) and \( w_3 \)) relative to the plain PCC mixture. This again demonstrates the ductile nature of the RAP-PCC material. The kink point stress ratio, \( \psi \), turns out to be constant when substituting Equations (8) and (9) into Equation (12):

\[
\psi = 1 - \frac{\text{CTOD}_f}{2G_f} = 1 - \frac{\text{CTOD}_f \times 1.4705 \times (K_f)^{1.5}}{2 \times (K_f)^{1.5} / E} = 0.265
\]

(30)

This current study might be the first one to show that a constant \( \psi \) should be used for the bilinear softening curve, despite a considerable amount of the exiting works which attempted to get a best estimate of \( \psi \) from different experimental approaches [26].

With all the parameters calculated in Table 7, the bilinear softening curves for the studied mixtures are plotted in Fig. 14. These curves are inputs for the numerical modeling of RAP-PCC fracture behaviors in the future study.

5.6. Comparison between the SEN(B) and the SCB test

A limited amount of single-edge notched beam specimens was tested based on the RILEM method described in Section 1.2.1. Five beams of 450 mm in length, 50 mm in thickness, and 100 mm in height were cast in the lab using the 0.40,520,40BRY mixture followed by a 28-day moisture curing in the standard curing room. The initial notch length was 33 mm and the testing beam span length was 400 mm.

The SEN(B) results are used to directly compare with the SCB test results. The comparisons of various parameters obtained from the two tests are summarized in Table 8. From Table 8, the averaged values obtained from the SCB test are invariably lower than those from the SEN(B) test for all the parameters. Interestingly enough, except for the \( \text{CTOD}_f \) and Q, all the other parameters show around 30% reduction when comparing the SCB test results with the SEN(B) test results. This 30% reduction might be the intrinsic coefficient between the newly developed SCB test and the more sophisticated SEN (B) test due to the size and geometry difference. On the other hand, the \( \text{CTOD}_f \) and Q values are comparable between the two tests; the statistical evaluation indicates that there is no sufficient evidence shows the results are different, which means the \( \text{CTOD}_f \) and Q value might be considered as size independent material properties. Any further discussions concerning the comparison of the SEN(B) and the SCB fracture tests need significant future research efforts. It is not the scope of this paper to investigate this aspect in detail.

6. Conclusions

While RAP-PCC has reported to have equivalent or even better
toughness, ductility, and fracture properties than conventional PCC, the available data on RAP-PCC’s fracture properties is limited. Besides, the existing test methods using a single-edge notched beam or a disk-shaped compact tension geometry suffer from some limitations. Therefore, an easy but effective method to characterize the fracture properties of the RAP-PCC needs to be developed. This paper presented an innovative fracture test using the SCB geometry to characterize the TPFP of the RAP-PCC. The important findings regarding the test development and the properties of the RAP-PCC are summarized as below:

- The SCB specimen with a 76 mm radius and a 38 mm notch length (A/R = 0.5) was selected as a feasible specimen dimension for an effective SCB fracture test when the maximum aggregate size is within 25 mm. The specimen with the 0.16 A/R ratio suffered from a sudden failure after the peak load, as such applying an unloading process when the load was still within 95% of the maximum load was not possible.
- The comparisons of the means of the fracture properties indicate that the RAP-PCC mixtures have similar $K_{IC}$, but higher CTOD, $G_C$, and $G_F$ relative to the plain PCC. An evaluation of the data through statistical approaches shows the $K_{IC}$ and $G_F$ for different mixtures are statistically similar, but the CTOD and $G_C$ for the RAP-PCC are statistically higher than the plain PCC.
- The theoretical tensile strength, $f_t$, of the studied mixtures is determined using the tested fracture properties and then compared with the conventional measurements from the flexural and splitting tensile tests. It is confirmed that the conventionally measured PCC tensile strength, $f_t'$, overestimates the $f_t$ of PCC.
- The material length, $Q$, of the PCC containing BRY RAP is statistically higher than that of the plain PCC, indicating that RAP-PCC is a more ductile material.
- The MOE determined from the SCB fracture test is invariably lower than that obtained from the conventional compression test, despite the fact the MOE from the SCB test is notch size dependent. It is hypothesized that tensile tangent MOE of PCC is smaller than compressive tangent MOE of PCC based on this finding.
- The bilinear softening curves for the studied mixtures are successfully generated using the SCB fracture test for future fracture behavior modeling. The RAP-PCC mixtures have higher displacement parameters relative to the plain PCC, which again demonstrates the ductile nature of RAP-PCC material. For the first time, the kink point stress ratio in the bilinear curve, $\psi$, is theoretically determined as a constant value 0.265.
- The direct comparison of the SCB results with the SEN(B) results indicates that the CTOD, and $Q$ value might be considered as size independent material properties. A 30% reduction might be the intrinsic coefficient associated with the size and geometry difference when comparing the SCB test with the SEN (B) test for $E$, $K_{IC}$, $G_C$, and $G_F$.

Declarations of interest

None.

Funding

This work was supported by the Texas Department of Transportation [Project number 0-6855].

<p>| Table 7 |
| Parameters for the bilinear softening curve. |</p>
<table>
<thead>
<tr>
<th>Mix type</th>
<th>$E$ (GPa)</th>
<th>$\nu^\prime$</th>
<th>$w_1$ (mm)</th>
<th>$w_a$ (mm)</th>
<th>$w_k$ (mm)</th>
<th>$f_t$ (MPa)</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40,520,REF</td>
<td>33.0</td>
<td>0.151</td>
<td>0.0132</td>
<td>0.0097</td>
<td>0.2459</td>
<td>2.28</td>
<td>0.265</td>
</tr>
<tr>
<td>0.40,520,40HOU</td>
<td>24.5</td>
<td>0.176</td>
<td>0.0247</td>
<td>0.0181</td>
<td>0.2647</td>
<td>2.10</td>
<td>0.265</td>
</tr>
<tr>
<td>0.40,520,40BRY</td>
<td>24.1</td>
<td>0.190</td>
<td>0.0332</td>
<td>0.0244</td>
<td>0.3588</td>
<td>1.86</td>
<td>0.265</td>
</tr>
</tbody>
</table>

* Poisson’s ratio was obtained according to ASTM C469 [44]. Detailed information can be found in the author’s previous publication [8].

<p>| Table 8 |
| Comparison of the SEN(B) test and the SCB test results for the 0.40,520,40BRY mixture. |</p>
<table>
<thead>
<tr>
<th>Specimen geometry</th>
<th>$E$ (GPa)</th>
<th>$K_{IC}$ (MPa m$^{1/2}$)</th>
<th>CTOD(mm)</th>
<th>$G_C$ (N/m)</th>
<th>$G_F$ (N/m)</th>
<th>$Q$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEN(B)</td>
<td>28.04 (8%)</td>
<td>0.996 (11%)</td>
<td>0.02450 (28%)</td>
<td>43.6 (22%)</td>
<td>152.4 (35%)</td>
<td>319 (37%)</td>
</tr>
<tr>
<td>SCB</td>
<td>15.00 (9%)</td>
<td>0.674 (9%)</td>
<td>0.0244 (21%)</td>
<td>30.4 (13%)</td>
<td>106.9 (24%)</td>
<td>303 (43%)</td>
</tr>
</tbody>
</table>

Di *ference −35% | −32% | −4% | −30% | −29% | −5% |

Note: The difference is calculated as the value difference normalized by the SEN(B) value.

* Suggests statistically significant comparisons.

Fig. 14. Bilinear softening curves for the studied mixtures.

Fig. 14. Bilinear softening curves for the studied mixtures.

Table 8

<table>
<thead>
<tr>
<th>Specimen geometry</th>
<th>$E$ (GPa)</th>
<th>$K_{IC}$ (MPa m$^{1/2}$)</th>
<th>CTOD(mm)</th>
<th>$G_C$ (N/m)</th>
<th>$G_F$ (N/m)</th>
<th>$Q$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEN(B)</td>
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<td>43.6 (22%)</td>
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</tr>
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<td>0.0244 (21%)</td>
<td>30.4 (13%)</td>
<td>106.9 (24%)</td>
<td>303 (43%)</td>
</tr>
</tbody>
</table>

Di *ference −35% | −32% | −4% | −30% | −29% | −5% |

Note: The difference is calculated as the value difference normalized by the SEN(B) value.

* Suggests statistically significant comparisons.
Acknowledgement

The research presented in this paper was partially supported by the Texas Department of Transportation and the Texas A&M Transportation Institute. Any opinions, findings, conclusions, and recommendations expressed in this paper are those of the authors alone and do not necessarily reflect the views of the sponsoring agencies. The authors greatly acknowledge Professor John Dempsey from Clarkson University for his assistance with the SCB geometry TPFP derivation.

Appendix A. SCB geometry TPFP derivation

This appendix presents a step by step procedure to get two-parameter fracture properties from the developed SCB fracture test.

Calculation of $K_{IC}$ and $CTOD_c$ based on LEFM

Weight function method

The weight function method is considered a versatile method to determine the stress intensity factor and the associated crack opening displacement. The weight function method states that once a two-dimensional elastic crack solution (reference solution) as a function of the crack length $A$ for any loading condition is known, the stress intensity factor for the same geometry being subject to any other loading can be obtained [46]:

$$K(A) = \frac{E'}{E} \int_0^A \sigma(Y) \frac{\partial U(A, Y)}{\partial A} dY$$  \hspace{1cm} (A.1)

Here $E' = E$ for plane stress condition and $E' = \frac{E}{1-\nu^2}$ for plane strain condition

$K_c(A)$ is the stress intensity factor for the reference solution

$U(A, Y)$ is the crack face displacement for the reference solution

$\sigma(Y)$ is the crackline stress

For convenience, the non-dimensional notation is used for the studied semicircular geometry (Fig. A1):

$$y = \frac{Y}{R}$$  \hspace{1cm} (A.2)

$$a = \frac{A}{R}$$  \hspace{1cm} (A.3)

$$u_i = \frac{U_i(A, Y)}{R}$$  \hspace{1cm} (A.4)

Specifically,

$$f_i(a) = \frac{K_c(A)}{\sigma(\sqrt{\pi A})}$$  \hspace{1cm} (A.5)

$$V_i(a) = \frac{E}{\sigma A} U_i(A, 0)$$  \hspace{1cm} (A.6)

Define

$$\frac{K(A)}{\sqrt{R}} = k(a) = \int_0^a \sigma(y) h_i(a, y) dy$$  \hspace{1cm} (A.7)

In a non-dimensional form:

$$K(A) = f(a) \sigma \sqrt{\pi A R}$$  \hspace{1cm} (A.8)
\[ f(a) = \int_{0}^{a} \frac{\sigma(y) h_{i}(a, y)}{\sqrt{\pi a}} dy \]  
(A.9)

In which the weight function is expressed [47]:

\[ h_{i}(a, y) = \frac{1}{\sqrt{2\pi a}} \sum_{i=1}^{5} \mathcal{G}(a) \left( 1 - \frac{y}{a} \right)^{\left( \frac{3}{2} \right)}, \quad y \leq a \]  
(A.10)

Where

\[ \mathcal{G}(a) = 2.0 \]  
(A.11)

\[ \mathcal{B}_{1}(a) = 6 \frac{(1-a)^{3/2}}{(1-a)} + 105E(a) \frac{aF(a)}{\sqrt{1-a}} + 4 \frac{aF(a)}{\sqrt{1-a}} - 30 \]  
(A.12)

\[ \mathcal{B}_{2}(a) = \frac{S_{1}(a)}{(1-a)^{3/2}} - 26 \frac{539E(a)}{\sqrt{1-a}} + 52a \frac{F(a)}{3F(a)} + 86 \]  
(A.13)

\[ \mathcal{B}_{3}(a) = \frac{S_{1}(a)}{(1-a)^{3/2}} + 154 \frac{539E(a)}{\sqrt{1-a}} + 308a \frac{F(a)}{5F(a)} - 434 \frac{3F(a)}{5} \]  
(A.14)

\[ \mathcal{B}_{4}(a) = \frac{S_{1}(a)}{(1-a)^{3/2}} - 54 \frac{189E(a)}{\sqrt{1-a}} - 36a \frac{F(a)}{5F(a)} + 144 \frac{3F(a)}{5} \]  
(A.15)

And

\[ E(a) = \frac{3\pi \Phi(a) - V(a)}{8\sqrt{2} F(a)} \]  
(A.16)

\[ S_{1}(a) = \frac{35}{4\sqrt{2}} \left[ 3\pi F(a) - \frac{\dot{V}(a)}{F(a)} \right] \]  
(A.17)

\[ S_{2}(a) = \frac{7}{2\sqrt{2}} \left[ -12\pi F(a) + 5 \frac{\dot{V}(a)}{F(a)} \right] \]  
(A.18)

\[ S_{3}(a) = \frac{9}{4\sqrt{2}} \left[ 7\pi F(a) - 3 \frac{\dot{V}(a)}{F(a)} \right] \]  
(A.19)

\[ \dot{V}(a) = (1+a)V(a) + a(1-a)V'(a) \]  
(A.20)

The non-dimensional crack opening displacement is expressed as:

\[ u(a, y) = \frac{\int_{0}^{y} k(s) h_{i}(s, y) ds}{E} \]  
(A.21)

The reference solution for semicircular geometry

Based on a previous work using finite element method [38], the reference solution for which a uniform crack face pressure is chosen can be obtained as below:

\[ f_{r}(a) = \frac{F(a)}{(1-a)^{3/2}} \]  
(A.22)

\[ V_{r}(a) = \frac{V(a)}{(1-a)^{2}} \]  
(A.23)

Where

\[ F(s) = \sum_{i=0}^{\infty} \alpha_{i} s^{i} \]  
(A.24)

\[ V(s) = \sum_{i=0}^{\infty} \gamma_{i} s^{i} \]  
(A.25)

The values for the coefficients \( \alpha_{i} \) and \( \gamma_{i} \) are presented in Table A.1.
Values for the coefficients $\alpha_i$ and $\gamma_i$.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_i$</th>
<th></th>
<th>$\gamma_i$</th>
</tr>
</thead>
<tbody>
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<td>1.1215</td>
<td>2.9086</td>
<td></td>
</tr>
<tr>
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<td>-4.3767</td>
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<td>7.9753</td>
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<td>-33.8874</td>
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</tr>
<tr>
<td>4</td>
<td>6.4138</td>
<td>64.2510</td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td>-83.8393</td>
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</tr>
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<td>68.1176</td>
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</tr>
<tr>
<td>7</td>
<td>3.1024</td>
<td>-22.8548</td>
<td></td>
</tr>
</tbody>
</table>

The expression for $\Phi(u)$ for the SCB reference case is written:

$$\Phi(u) = \sum_{i=0}^{7} \kappa_i s^i$$

(A.26)

Where the coefficients $\kappa_i$ ($i = 0, 1, \ldots, 7$) are 0.6289, -1.081, 3.5188, -5.8425, 6.6906, -5.6382, 3.3323, and -0.9800, respectively.

Semicircular bending crackline stress

According to Adamson et al. [38], the SCB crackline stress $\sigma_y$ is expressed as:

$$\sigma_y = \sigma_{SCB} = \sigma_{SCB} \sum_{i=0}^{N} c_i y^i$$

(A.27)

Where

$$\sigma_{SCB} = \frac{P}{BR}$$

(A.28)

The fitting coefficients are listed in Table A.2 for different $S/R$.

### Table A.2

<table>
<thead>
<tr>
<th>$S/R$</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
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</thead>
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<td>16.443</td>
<td>-15.776</td>
<td>5.484</td>
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<tr>
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<td>1.792</td>
<td>-7.227</td>
<td>12.026</td>
<td>-9.981</td>
<td>3.122</td>
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<td>2.072</td>
<td>-7.155</td>
<td>9.090</td>
<td>-6.163</td>
<td>1.658</td>
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<tr>
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<td>2.365</td>
<td>-7.086</td>
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<td>-1.957</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td>2.702</td>
<td>-7.852</td>
<td>5.679</td>
<td>-1.443</td>
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</tr>
<tr>
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<td>3.043</td>
<td>-8.503</td>
<td>5.153</td>
<td>-0.802</td>
<td></td>
</tr>
</tbody>
</table>

Determination of $K_i$ and CTODc from the load-CMOD curve

In the load-CMOD curve, the CMOD $w(A) \equiv 2U(A,0)$.

$$V(a) = \frac{EBR}{PA} U(A,0) = \frac{EBR}{PA} w(A)$$

(A.29)

$$w(A) = \frac{2V(a)}{E BR}$$

(A.30)

In the load-CMOD curve, the compliance $C$ are calculated through the linear portion (within 10% and 40% of the peak load) of the loading and unloading step.

$$C = \frac{w(A)}{P}$$

(A.31)

The modulus of elasticity is expressed as:

$$E = \frac{V(a)A}{BCR} = \frac{2V(a)a}{BC}$$

(A.32)

Define $S(a) = V(a)a$,

$$E = \frac{2S(a)}{BC}$$

(A.33)

Using the initial compliance $C_i$ in the loading portion of the curve, the modulus of elasticity is written:

$$E = \frac{2S(a_i)}{BC_i}$$

(A.34)

For the unloading compliance $C_u$, the modulus of elasticity is written:

$$E = \frac{2S(a_u)}{BC_u}$$

(A.35)
Assume E doesn’t change during loading and unloading process:
\[
2S(a_c) = \frac{2S(a_c)}{BC_a} \quad BC_a \quad S(a_c) = \frac{C_a}{C} S(a_c) \quad (A.36)
\]

The effective-elastic cracking length ratio \( a_c \) can be easily determined through the Solver function in the Excel or using the numerical computing software such as Maple.

Once the \( a_c \) is determined, the critical fracture toughness is computed by:
\[
K(A) = \sqrt{R} k(a_c) = \sqrt{R} \int_0^{a_c} \sigma(y) h_s(a_c,y) \, dy 
\]

And the critical cracking tip opening displacement is written as:
\[
\text{CTOD} = 2U(a_c, a_0) = \frac{2R}{E} \int_{a_c}^{a_0} \kappa_s h_s(a_0) \, ds
\]

References


