Diffractive optical elements as raster-image generators

Matthias Gruber

The use of diffractive optical elements (DOEs) to generate complex raster images for a primarily artistic purpose is dealt with. Aspects of human vision that are relevant for the design of such elements are discussed. A design method based on an iterative Fourier transform algorithm and extended with elements from the direct-binary-search and the simulated-annealing algorithms is described. The proposed method provides a large set of parameters that can be adjusted freely to optimize it for any given design task. For demonstration a phase-only DOE was designed that generates an image of a Chinese dragon as a diffraction pattern. It was realized as a surface-relief element on a planar substrate through multilevel binary lithography and reactive-ion etching. Experimental tests confirm the usefulness of the design and the fabrication procedures to achieve excellent image quality. © 2001 Optical Society of America

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1. Introduction

Over the past three decades the design and fabrication methods for (computer-generated) diffractive optical elements (DOEs) have been refined to such a degree that it has become possible to manipulate light almost arbitrarily with such elements. It is therefore no surprise that the range of applications for DOEs is growing fast. A large part of the various applications described in the more recently published literature about diffractive optics1–3 requires beam splitting or beam shaping, i.e., the transformation of a light beam with certain (known or unknown) characteristics into one with a desired spatial phase or amplitude distribution or both. In a wider sense such a transformation can be interpreted as the generation of a certain desired image.

Naturally, the potential to generate almost any diffraction pattern with a DOE can also be and has been used in an artistic way to produce high-quality images with nice visual appeal. Although there is presumably not much economic potential in artistic applications for DOEs, the playful use of diffractive optics provides valuable insight into optical technology, and it is a good way to demonstrate its potential to a nonexpert audience. This was one of the reasons for the organizers of the Diffractive Optics and Micro-Optics 2000 (DOMO 2000) conference series4 to introduce and host a so-called diffractive beauty contest in which contestants were invited to be creatively artistic. Needless to say, this contest instantly became a conference highlight.5,6

To obtain DOEs that generate complex arbitrary images, one can make use of well-established and readily accessible design tools (cf. Refs. 1–3 and 7, and references therein). However, to make optimal use of the available resources, one should pay attention to the physiological characteristics of human vision and adapt the design algorithms accordingly. That is the focus of this paper. We discuss a method that is based on an iterative Fourier transform algorithm8,9 (IFTA) and is extended with elements from the direct-binary-search10 (DBS) and the simulated-annealing11 (SA) algorithms to design periodic phase-only DOEs. We also describe the fabrication procedure for realizing them as surface-relief phase elements on planar substrates.

The paper is intended to serve as a handbook for people with a practical interest in DOEs for artistic applications. It provides the theoretical background and an algorithmic toolkit to design periodic DOEs that generate large $[O(10^3 \times 10^3)]$ arbitrary raster images in a setup like the one shown in Fig. 1. The rastering is a consequence of the periodicity demand that we impose on the DOE. If the design is properly adapted to the physiology of human vision this condition does not limit the subjective image quality at all. However, it greatly simplifies the de-
sign problem and ensures a good result for virtually any desired diffraction pattern, as is explained in detail in Section 2. Sections 3 and 4 provide the mathematical basis and a description of the DOE design procedure, respectively. In Section 5 this procedure is demonstrated for the element with which we participated in the DOMO 2000 diffractive beauty contest. Section 6 is about the fabrication of our demonstrator DOE and about experimental results. Section 7 provides a conclusion and an outlook.

2. Raster Approach and Visual Perception

Raster images are, by definition, composed of regular arrays of picture elements or pixels. Pictorial information can be coded in various features of the pixels, such as their color or brightness, and pixels may be simple uniform patches of a certain gray level or have a more complex structure and even constitute small images themselves.

Rastering is familiar to the general public because it is the standard method of displaying pictorial information in printed media from books to newspapers to advertising billboards and on TV screens and computer monitors. In CRTs the raster is typically defined by a metal shadow mask that is placed just behind the phosphor-coated front screen and in flat-panel displays by the geometry of liquid-crystal cells; information is coded through the brightness that each pixel displays. In printed media usually dot-width modulation is used as the coding scheme; it is a binary method in which different hues and brightness levels are implemented through dots of different size. For black-and-white images only black dots are needed; for color images usually three elementary colors (cyan, magenta, and yellow) are used in addition to black.

With clever exploitation of the characteristics of visual perception it is possible to raster-print any image in such a way that a human observer does not notice its discrete nature. The key is to make the pixels appear small enough that their angular frequency surpasses the cutoff frequency of the human eye, which is approximately 30 line pairs/deg. Then the image will look smooth and continuous. Surpassing the cutoff angular frequency is equivalent to different minimum required pixel densities in different print media depending on the typical observation distance. In newspapers, for example, densities corresponding to approximately 100 lines/in. (lpi) are common.

The fact that a pixel raster is not perceived if it is fine enough turns out to be very useful for our DOE design task because it allows us to apply discrete algorithms without having to cut back on the (subjectively achievable) image quality. We opted for an IFTA-based procedure because the IFTA has proved to be an extremely powerful design approach that can easily be optimized for a variety of different design problems.

Phase-only DOEs, which are also termed kinoforms, are the preferred DOE type considered in this paper not only because they do not waste energy through absorption but also because accurate and reliable manufacturing methods are available.
We focus on a realization in the form of stepwise constant surface-relief elements (staircaselike elements) that can be fabricated with high yield on planar substrates by means of multilevel binary lithography and reactive-ion etching.

One more aspect concerning the physiology of vision needs to be pointed out here, namely, the importance of contours for the perception of shape, hence for the recognition of patterns in a scene. Mathematically contours are related to the second derivative of brightness. Interestingly, they need not be continuous; thanks to the brain's excellent capability to compensate for missing details it can easily detect contours that are formed by edge segments or points. For DOE design it is important that contours constitute only a small but particularly important fraction of the parameters of an image. This concept is relevant in the context of priorities presented in Section 5.

3. Mathematical Model of Periodic Diffractive Optical Elements

An IFTA-type design algorithm, as envisaged in this paper, is based on a scalar paraxial modeling of optical diffraction. In this section, we briefly recapitulate the respective mathematical formalism, thereby following the nomenclature of Ref. 19, which can be traced back to Goodman's well-known introductory book on Fourier optics. We also discuss the applicability of a scalar paraxial model.

Because we focus on surface-relief phase-only DOEs with a maximum relief depth of the order of micrometers, the Kirchhoff approximation is justified, i.e., the optical effect of the DOE can be expressed through a simple multiplication of its complex transmission function $t(x, y)$ with the illuminating light distribution $s(x, y)$, resulting in $g(x, y)$. If periodicity is assumed, $g(x, y)$ can be expressed as

$$g(x, y) = t(x, y)s(x, y) = \left[\text{comb}(x, y) \otimes p(x, y)\right]s(x, y), \quad (1)$$

where $\otimes$ symbolizes the convolution operation and $p(x, y)$ represents a single period. Assuming that $p(x, y)$ consists of an array of $N \times M$ rectangular pixels (see Fig. 2) means that $g(x, y)$ can be rewritten as

$$g(x, y) = \left[\text{comb}(x, y) \otimes w(x, y) \otimes e(x, y)\right]s(x, y), \quad (2)$$

$$w(x, y) = \sum_{n=1}^{N} \sum_{m=1}^{M} \exp(i\phi_{nm})\delta(x - n/N, y - m/M), \quad (3)$$

$$e(x, y) = \text{rect}(Nx)\text{rect}(My). \quad (4)$$

Equation (3) defines an $N \times M$ array of complex-valued weighting factors that carry the information about the phase shifts $\phi_{nm}$ that are induced by each pixel of the DOE, whereas Eq. (4) describes their geometry. If we further assume orthogonal unit-amplitude plane-wave illumination of $P \times Q$ periods of the DOE, $s(x, y)$ is reduced to a simple flat top

$$s(x, y) = \text{rect}(x/P)\text{rect}(y/Q), \quad (5)$$

and the far-field diffraction pattern that is obtained through a Fourier transformation of $g(x, y)$ takes in the form of

$$G(u, v) = \mathcal{F}\{g(x, y)\} = \left[\text{comb}(u, v)W(u, v)E(u, v)\right] \otimes S(u, v), \quad (6)$$

where

$$W(u, v) = \mathcal{F}\{w(x, y)\} = \frac{1}{NM} \sum_{n=1}^{N} \sum_{m=1}^{M} \exp(i\phi_{nm})$$

$$\times \exp\left[i2\pi\left(\frac{nu}{N} + \frac{mv}{M}\right)\right], \quad (7)$$

$$E(u, v) = \mathcal{F}\{e(x, y)\} = \text{sinc}(u/N)\text{sinc}(v/M), \quad (8)$$

$$S(u, v) = \mathcal{F}\{s(x, y)\} = PQ \text{sinc}(Pu)\text{sinc}(Qu). \quad (9)$$
The diffraction orders defined by the comb function in Eq. (6) are weighted with \( W(u, v) \), which has a periodicity of \( N \) and \( M \) in \( u \) and \( v \), respectively, and the whole diffraction pattern is modulated by an envelope function \( E(u, v) \) that is determined by the pixel geometry of the DOE. For the rectangular pixels of Eq. (4) a sinc function with zero crossings at orders that are integer multiples of \( N \) and \( M \), respectively, is obtained.

Support of \( G(u, v) \) at noninteger spatial frequencies is due to the convolution with the function \( S(u, v) \). If a sufficiently large number of periods (five or more in each dimension, as a rule of thumb) of the DOE is illuminated, \( S(u, v) \) is compressed well enough to prevent significant overlap with neighboring orders, and the diffraction pattern appears to consist of discrete orders, the shape of each of which is defined by \( S(u, v) \). Apparently, the Fourier approach has the welcome side effect of making raster-image generation tolerant with respect to varying experimental conditions: A change in the illumination characteristics that affects the function \( s(x, y) \) has influence on only the spot shape \( S(u, v) \) but not the weighting factors \( W(u, v) \).

It remains to be discussed whether a scalar paraxial theory is sufficient to model the physical reality. A comparison of our assumed projection geometry (Fig. 1) with conditions given by Goodman (Ref. 20, Chap. 4) shows that we operate in the Fresnel regime but not in the Fraunhofer regime, as is required for the Fourier approach. However, as we see in Section 5, this problem can easily be overcome with a trick that is routinely used in the lab, that is, the use of a lens to simulate infinity at a finite distance.

4. Design Algorithm

The essence of DOE design is to find the functions \( g(x, y) \) and \( G(u, v) \) that are a Fourier pair and satisfy the given constraints for both domains. For that purpose an IFTA-based algorithm is applied here that has been extended by elements from the DBS and SA. As a numerical procedure, it needs to operate on finite quantities instead of the continuous \( g(x, y) \) and \( G(u, v) \). We use two \( N \times M \) matrices \( w_{ij} \) and \( W_{kl} \) that are obtained through a sampling of \( w(x, y) \) and \( W(u, v) \) at values \( (x = i/N, y = j/M) \) and \( (u = k, v = l) \), hence that form a discrete Fourier pair.

Before the iterative design process can start the constraints that apply to \( g(x, y) \) and \( G(u, v) \) have to be reformulated for \( w_{ij} \) and \( W_{kl} \). For a phase-only DOE with a desired intensity pattern \( I_d(u, v) \) and complete phase freedom in the Fourier domain the following two conditions are obtained:

\[
\begin{align*}
\phi_{ij} &= \arg(w_{ij}) = \arg(\tilde{w}_{ij}) \\
I_{d}(k, l) &= \frac{\left|I_{d}(k, l)\right|^2}{\left|E(k, l)\right|^2},
\end{align*}
\]

where \( \phi_{ij} \) and \( A_{kl} \) symbolize the moduli of \( w_{ij} \) and \( W_{kl} \), respectively, and \( C \) is a constant. The IFTA routine is outlined in the flow chart of Fig. 3 and involves two additional matrices \( \tilde{w}_{ij} \) and \( W_{kl} \) that symbolize the situations before and after the adaption to the constraints in the respective domains. The sequence of Fourier transformation and adaption is iterated until the algorithm converges; in the ideal case this means identity between \( w_{ij} \) and \( \tilde{w}_{ij} \) and between \( W_{kl} \) and \( W_{kl} \), respectively.

The iteration may start at any corner of the flow chart. Here a quadratic distribution of the form

\[
\phi_{ij} = \arg(w_{ij}) = p_1 i + p_2 i^2 + p_3 j + p_4 j^2 \quad (12)
\]

in the upper left-hand corner is chosen as the initial state; parameters \( p_1 - p_4 \) are adjusted in such a way that the resultant \( |W_{kl}|^2 \) approximately covers the main region of interest of the raster image that is to be generated (cf. Fig. 5, below). We dispense with more sophisticated methods to construct an initial state because the extension of the pure IFTA with elements from SA, as discussed below, ensures that a near-optimal solution will be found even if the initial state is suboptimal. In comparison with a random initial distribution the quadratic approach has the advantage of being speckle free, which is essential if the phase profile is eventually to be unwrapped.

In accord with classical adaption rules, the amplitudes of \( W_{kl} \) and \( \tilde{w}_{ij} \) have to be discarded and replaced with the desired ones in each adaption step, while the phase values are kept. We apply this rule in only the space domain through

\[
w_{ij} = C \exp[i \ \arg(\tilde{w}_{ij})].
\]

For the frequency domain the following modified rule has been developed that provides better control over the solution that the algorithm converges toward:

\[
W_{kl} = \alpha_{kl} A_{kl} \exp[i \ \arg(W_{kl})] + (1 - \alpha_{kl}) W_{kl}. \quad (14)
\]

The \( \alpha_{kl} (0 < \alpha_{kl} \leq 1) \) are variable adaption parameters that may depend on both the desired amplitudes \( A_{kl} \) and the current amplitudes \( |W_{kl}| \); the optimal definition is problem related.

The approach of Eq. (14) allows one to set priorities in the design process. Let us consider the example of an array generator, i.e., a DOE that should diffract as much of the intensity of an incoming light beam as possible into an array of equi-intense diffraction orders. With the classical adaption rule, \( \alpha_{kl} = 1 \), \( \forall k, l \), the diffraction efficiency, i.e., the ratio of the cumulated intensity of all the array spots and the
intensity of the incoming beam, is usually maximized. The uniformity of the array, however, is often rather poor (peak-to-valley fluctuations of the order of 10% or more). In contrast, if $\alpha_{kl}$ is set to 1 for all spatial frequencies that make up the array and to considerably lower than 1 otherwise, thus assigning top priority to the array frequencies, then nearly perfect uniformity will be achieved at the cost of only a slightly lower diffraction efficiency. In other words, the conflicting design goals of high diffraction efficiency and good uniformity can be balanced by one's appropriately setting the adaption parameters. The concept of priorities as outlined above can be considered as a generalization of the approach of using dummy areas to design kinoforms.

Fienup has already pointed out that the IFTA can be interpreted as a gradient-descent method that leads to a solution that constitutes a local minimum of some error function. The IFTA is neither capable of finding the global minimum with certainty nor of providing information about how close the solution obtained is to the globally optimal solution. SA, which doesn't suffer from this disadvantage and has been successfully applied for DOE design before, however, requires a huge amount of computation before the algorithm settles in the globally optimal solution. Therefore some kind of combination that is not as computation intensive as the pure SA and can overcome an IFTA-typical stagnation in a local minimum of the error function appears attractive.

Our suggestion is to start with an IFTA iteration cycle, as discussed above. The intermediate solution $w_{ij}^{\text{int}}$ is then subjected to the procedure outlined in Fig. 4. A small fraction of the matrix elements is randomly selected, and their phase values are randomly altered, yielding a modified matrix $w_{ij}^{\text{mod}}$ that is then used as the start distribution for another optimization through the IFTA. If the iteration cycle yields a better solution $w_{ij}^{\text{end}}$ than the intermediate solution, then $w_{ij}^{\text{end}}$ is memorized and used as a new intermediate solution; otherwise $w_{ij}^{\text{mod}}$ and $w_{ij}^{\text{end}}$ are discarded. The criterion for deciding whether a solution is better can be freely chosen, for example, the diffraction efficiency. The whole procedure is repeated until no further improvement occurs for several successive trials.

This proposed algorithm resembles the DBS in the sense that random phase changes in the DOE structure are accepted when they improve the performance according to some evaluation criterion. And it resembles SA in the sense that phase changes that initially deteriorate DOE performance may still be accepted, thus overcoming stagnation in local minima. In fact, $w_{ij}^{\text{mod}}$ will almost always perform worse than the intermediate solution $w_{ij}^{\text{int}}$ from which it is derived, and an improvement will become evident only after the subsequent IFTA optimization.

We tested this algorithm and found that, by random alteration between 2% and 5% of the matrix elements $w_{ij}^{\text{int}}$ and by the running of approximately 100 trials, a typical improvement in terms of diffraction efficiency of almost 4 percentage points with respect to the initial IFTA cycle could be achieved. In Section 5 the algorithm is demonstrated for the generation of a large and complex raster image.

5. Design Example

The reason for designing a large artistic raster image was our participation in the DOMO 2000 diffractive beauty contest. In accord with Optical Society of America guidelines, entries of the artistic category of the contest had to demonstrate their DOE’s at the conference and be judged on the basis of the visual appeal of the generated image. Because the year 2000 happened to be a dragon year according to the Chinese zodiac, we decided to use a dragon as the motif of our DOMO 2000 entry.

A drawing of a Chinese dragon in cartoon style was transformed into a gray-level raster pattern consisting of $500 \times 500$ pixels. The space-bandwidth product is therefore comparable with that of a TV screen and ensures good reproduction of image details and smooth contours. Unlike on a TV screen or a computer monitor, however, where pixels are arranged in horizontal and vertical lines, our raster image was rotated by $45^\circ$; in this way, we can exploit the physical effect that a raster of horizontal and vertical dots that is barely perceived as discrete by a human observer is no longer resolved and thus appears continuous after a $45^\circ$ rotation. The technique of one’s using such skewed dot rasters is routinely employed...
for image setting in printed media\cite{14}; we implement it by rotating the whole DOE by 45° during reconstruction. As a consequence, the figure of the dragon has to be counter-rotated in the raster image that defines the desired diffraction pattern so that it appears upright during reconstruction.

Our target diffraction pattern (which is not shown explicitly but looks essentially the same as the last snapshot in the series of Fig. 5) contains four different brightness levels $b$, namely, the ones represented by the eyes ($b = 1$), the nose ($b = 0.55$), and most parts of the body ($b = 0.35$) of the dragon as well as the background ($b = 0$). These values are the desired intensities $I_d(u, v)$ for Eq. (11).

Some attention needs to be paid to an appropriate setting of the adaption parameters $\alpha_{kl}$. It was pointed out earlier that contours play a crucial role in visual perception; they should therefore receive the

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Fig. 5. Several stages in the design process of the dragon DOE: $\eta$ is the numerically obtained diffraction efficiency; the SNR is defined as the ratio of the average intensity of all diffraction orders that are part of the body of the dragon and the highest intensity anywhere in the background.
highest priority. The second most important objective is a uniform dark background without noisy hot spots. The uniformity of the spots that make up the body of the dragon is less important because the human eye tolerates comparatively large fluctuations. These priorities were implemented by our setting $\alpha_{kl} = 1$ for the contours of the dragon and $\alpha_{kl} = 0.2$ for the bright patches of the body. For the background pixels, we used a range of values between 0.2 and 0.8 and the rule that a certain $\alpha_{kl}$ is set for the larger of the higher noise levels of pixel $(k, l)$ in the respective iteration; in this way noisy hot spots can be targeted selectively and attenuated efficiently.

With these parameters and a quadratic phase profile at the start the initial IFTA cycle yields an intermediate solution $u_{ij}^{\text{int}}$ that already diffracts more than 92% of the intensity into the diffraction orders that are part of the body of the dragon. In subsequent iteration steps this value could be improved to just greater than 95%. We ran 200 trials in which a constantly decreasing percentage of the matrix elements $u_{ij}^{\text{int}}$ was randomly altered, starting at 5% and ending at 2%. In the final matrix $W_{kl}^{\text{fin}}$ no individual background order surpasses 1.14% of the average intensity of the diffraction orders that compose the body of the dragon, which makes the background appear uniformly dark. And, although the intensity fluctuations of spots that make up the body of the dragon are of the order of 20%, they will hardly be noticed visually.

The computational effort was low enough to be manageable on a typical modern PC. In total, approximately 600 Fourier transformations of a complex-valued array of size $500 \times 500$ pixels, the adaptions to the constraints in both domains as well as some control and evaluation operations had to be carried out. The computational time of several hours that we actually needed is not representative because we used an interpreted MATLAB code that would run much faster.

Figure 5 gives a graphical impression of how the algorithm of Fig. 4 progresses toward the final solution. For six different stages the phase profile $\arg(u_{ij})$ and the corresponding (theoretical) diffraction pattern $|W_{kl}E(k, l)|^2$ are shown. The efficiency $\eta$ is defined as the ratio of the cumulated intensity of all the diffraction orders that compose the body of the dragon and the total intensity of the displayed image. The signal-to-noise ratio (SNR) is defined as the ratio of the average intensity of all the diffraction orders that compose the body of the dragon and the highest intensity anywhere in the background.

To obtain a (finite) periodic DOE the numerically optimized matrix $u_{ij}$ was replicated in both dimensions. Thereby we had to keep in mind that a maximum total area of $1 \text{ cm} \times 1 \text{ cm}$ was allowed for the DOE in the beauty contest. We used $5 \times 5$ periods to cover this area, which means that a single period measures $2 \text{ mm} \times 2 \text{ mm}$ and each of the $500 \times 500$ pixels per period has a size of $4 \mu \text{m} \times 4 \mu \text{m}$.

If a distance $L$ of 5 m between the DOE and the projection screen (cf. Fig. 6) is assumed the diameter $D$ of the diffraction pattern can be estimated as follows:

$$D = L \tan \beta = L \frac{\lambda}{N_e}.$$  \hspace{1cm} (15)

The denominator of expression (15) expresses the (linear) extension of one elementary cell ($N$ pixels of length $e$), whereas $\lambda$ stands for the optical wavelength that is used to illuminate the DOE, in our case 632.8 nm. Hence a value of approximately 0.8 m is obtained for $D$, which means that the 500 diffraction orders will have a mutual spacing of nearly 1.6 mm. This spacing corresponds to an angular frequency of more than 50 line pairs/deg and thus surpasses the cutoff frequency of the human eye. The displayed dragon therefore appears smooth and continuous.

One problem, however, remains to be solved. Without appropriate measures the diffraction orders do not form sharp spots on the observation screen. Instead, they are blurs of approximately the same size as the clear aperture of the DOE ($1 \text{ cm} \times 1 \text{ cm}$) so that the observed image quality is severely deteriorated as a result of overlap. To avoid this deterioration requires that a parabolic phase profile that implements a lens with 5-m focal length be added. With this measure the theoretically expected Fraunhofer diffraction pattern is generated on the observation screen. We combined such a parabolic phase profile and the optimized finite grating structure into one single DOE.

6. Fabrication and Test

The dragon DOE was designed as a spatially discrete (pixelated) structure with infinite phase resolution. For realization as a multilevel binary surface-relief element this phase profile needed to be quantized. To this end, we projected each phase value onto the nearest of eight equally spaced discrete values in the interval $[0, 2\pi]$. The combination with the parabolic lens profile thereby has the convenient side effect of effectively randomizing the quantization errors made for $g(x, y)$; in this way systematic reconstruction errors in the diffraction pattern are avoided.

To realize eight discrete phase levels required that three binary photolithographic masks be fabricated, which was done with e-beam technology. A fused-
silica substrate with a 2-in.- (5.08-cm-) diameter and a 1-mm thickness was then spin coated with photore sist (Allresist, AR P 3120), and the first mask pattern was transferred into the layer of photore sist by use of contact copying with a mask aligner (Suss, Model MA 4). After developing and hard-baking the photore sist on the substrate the binary pattern was transferred into the fused silica by use of reactive-ion etching with an Oxford Model Plasmalab 80 Plus dry-etching machine and a CHF3/Ar plasma. The (nominal) etch depth for the first layer is 173 nm, corresponding to a λ/8 wave-front delay. The procedure was repeated for the other two mask patterns with etch depths of 346 nm and 692 nm, corresponding to λ/4 and λ/2 wave-front delays, respectively. Finally, a 2-mm broad rim around the edges of the DOE was coated with aluminum to define the clear aperture precisely. Figure 7 shows the finished component, which was additionally masked with black paper.

In optical experiments this surface-relief phase-only DOE performs excellently. Figure 8 shows a picture of the central part of the diffraction pattern that depicts the dragon figure, which almost perfectly matches the theoretical expectations. Background noise is practically invisible, and the four different brightness levels can be distinguished well visually. The only apparent distinction from the calculated diffraction pattern defined by \( |W_{k,l}E(k, l)|^2 \) is the blur around the zero order (cf. the close-up in Fig. 9), which is due to small etch-depth errors.

The optical experiments also nicely confirm the theoretical predictions about the higher-order repeti tions of the generated raster image. In accord with Eqs. (6)–(9), the dragon figure is periodically repeated in the diffraction pattern and weighted with a slowly varying sinc function \( E(u, v) \) (except for the central image, of course, for which the sinc modulation has been compensated). These repetitions and the sinc modulation are clearly observable in Fig. 10; note that a square area in the central part of the (white) projection screen was covered with black paper for this recording to adapt the large intensity range in

Fig. 7. Finished dragon DOE on a fused-silica substrate.

Fig. 8. Central part of the experimentally observed diffraction pattern depicting a rastered dragon. The picture was recorded with a high-resolution 16-bit CCD camera.

Fig. 9. Close-up of the image of Fig. 8 showing the discrete nature of the diffraction pattern and the zeroth-order blur.
the diffraction pattern to the low dynamic range of the CCD camera.

7. Conclusion and Outlook

In this paper an algorithm for designing periodic DOEs for artistic applications has been described. It is based on the IFTA and extended with elements from the DBS and SA. Through a set of freely adjustable parameters it can be adapted to specific design problems. The good performance of this algorithm has been demonstrated with the design of a DOE that generates a large raster image of a Chinese dragon. The DOE has been realized as a surface-relief phase-only structure on a fused-silica substrate by multimask photolithography and reactive-ion etching.

An artistic application like our example of a raster image depicting a cartoon-style dragon is certainly spectacular and well suited to raise attention. However, it is not the only application for which our design algorithm is well suited. The field of optical document security,\textsuperscript{27} which has become increasingly important, can also benefit from it. Sinzinger\textsuperscript{28} proposed a system concept for optical authentication of documents like banknotes that is based on planar integration of free-space optics\textsuperscript{29} and optical correlation.\textsuperscript{30} This approach requires DOEs that generate specific complex diffraction patterns with high fidelity under real-world conditions. The methods described in this paper are an ideal tool for realizing such optical components.

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References