Modeling the heterogeneous hydraulic properties of faults using constraints from reservoir-induced seismicity

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Received 23 August 2004; revised 10 March 2005; accepted 11 May 2005; published 2 September 2005.

[1] This research uses observations of reservoir-induced seismicity beneath Açú Reservoir, NE Brazil, to investigate the spatial distribution of permeability within the damage zone surrounding faults. The Açú dam is a 34 m high earth-filled dam constructed in 1983 on an area of Precambrian shield. Our previous work has shown that fluctuations in seismic activity are related to varying reservoir level via the diffusion of pore pressure within high-permeability faults embedded in a lower-permeability matrix. High-resolution monitoring of the seismic activity within individual faults, using a network of three-component digital seismographs, has revealed a complex spatial pattern of earthquake clustering and migration that suggests heterogeneous fault zone hydraulic properties are present. We first review the laboratory and field evidence for variations in hydraulic properties associated with (1) structural architecture of faults and (2) confining pressure. We then model flow through a heterogeneous two-dimensional (2-D) fault embedded in, and explicitly coupled to, a 3-D medium and include a power law decay in diffusivity with depth associated with crack closure. Diffusivity of the fault is represented by a spatially correlated random field. We vary both the correlation length and variance of the diffusivity field and calculate the time lag between the maximum reservoir level and the maximum piezometric head in the depth range of observed seismic activity. By assuming that individual earthquake ruptures occur when the local piezometric head is at a maximum, we are able to infer the correlation length and variance that best explain the spatiotemporal pattern of the activity within each seismic cluster. The spatial and temporal evolution of seismicity within clusters is only found to be consistent with a causal mechanism of pore pressure diffusion when significant spatial structure is present in the heterogeneous fault hydraulic properties.


1. Introduction

[2] Understanding the role of faults in fluid flow and chemical transport is critical for the oil and gas, waste disposal and deep storage industries: Faults can traverse many lithological sequences, forming large-scale structures that span several kilometers laterally and over depth; their sheer physical extent implies that their hydraulic properties have a major influence on deep flow systems. Fault hydraulic properties vary considerably over both space and time (Hooper, 1991; Caine et al., 1996; Evans et al., 1997; Fairley et al., 2003). The temporal and spatial evolution of fault hydraulics is determined by the geomechanical properties of the host rock, the local and regional stress regime, the fluid pressure field and the geochemical environment controlling subsequent mineral deposition. The complex interaction of these factors over time results in observations of fault hydraulics that range from flow barriers to major conduits, with many faults containing individual sections of each. Consequently, they form one of the greatest components of uncertainty both in
models of oil and gas reservoir dynamics and in risk assessment models for waste disposal.

[1] Prior estimation of fault hydraulic properties without detailed field data is highly error prone. Even where field data are available, they are derived from limited outcrop and borehole exploration and cannot possibly adequately characterize the complex hydraulic structure of the entire fault plane. In this research, we show that reservoir-induced seismicity (RIS) data caused by pore pressure diffusion, such as that observed beneath water reservoirs, can provide an alternative data source for constraining hydraulic fault plane heterogeneity. The research presented here compares three-dimensional (3-D) fluid flow simulations and field observations of reservoir-induced seismicity (RIS) beneath Açú Reservoir, NE Brazil, to infer detailed information on the three-dimensional heterogeneous nature of fault hydraulic properties in the Açú region.

1.1. Induced Seismicity at Açú

[4] Açú Reservoir was constructed in 1983 and has a capacity of $2.4 \times 10^9$ m$^3$ maintained by a 34 m high earth-filled dam constructed on Precambrian shield. Annual reservoir level variation is 3–6 m which results in annual seismic activity due to pore pressure diffusion [Ferreira et al., 1995; do Nascimento et al., 2004, 2005]. A detailed analysis of the seismicity data at Açú is given by do Nascimento et al. [2004]. Next, we briefly describe only those characteristics of the RIS data that are directly relevant to determining patterns of fault heterogeneity.

[5] Seismic activity in the Açú area has been monitored over a 10 year period from 1987 to 1997. However, hypocentral information is only available for three discrete time periods (Figure 1a). Two field campaigns, using single-component smoked-drum recorders, were carried out during October to December 1989 and November 1990 to March 1991 [Ferreira et al., 1995]. These data have relatively large location errors (~0.5 km horizontal, poor vertical control). However, from 1994 to 1997, a network of three-component digital seismographs were operational which have provided a very accurate assessment of hypocentral locations during this latter period with errors of 100–300 m [do Nascimento et al., 2004]. Analysis of the data in Figure 1a has shown that seismicity occurs within well defined clusters, three of which (those active during 1994–1997) are marked on Figure 1a as clusters a, b, and c.

[6] Figure 1b shows the time series of daily water depth in the reservoir from 1987 to 1997. Also plotted is the frequency of seismic events for each month of this 10 year period. For the data presented here there is no correlation between earthquake magnitude and either hypocentral depth or time delay. However, we would expect some correlation between yield strength and diffusivity in fractured rock, i.e., areas of the fault that are more heavily fractured may be also weaker. As there is no known consistent relationship between these two properties, we have explored their combined effect through spatial variation of a single property, diffusivity. Therefore the research presented here uses numerical modeling of pressure diffusion through a hydraulically heterogeneous fault to predict the observed temporal and spatial scatter within seismic clusters at Açú, and hence to infer valuable information on the likely internal diffusivity (and possible yield strength) structure of fault planes in this region.

1.2. Theoretical Background

[7] To further investigate the role of pore pressure diffusion, Figure 1c shows a detailed spatiotemporal picture of the seismic evolution of cluster a (the most active cluster). Here, delay time from the previous reservoir peak is plotted against depth for all events within the cluster. This enables easy discrimination of events that must have a causal mechanism of pore pressure diffusion. For clarity, the reservoir level and a histogram of event frequency are also shown on Figure 1c. The only events that could possibly be attributed to an undrained response are those that occur between 270 and 330 days when the reservoir level is rising.

[8] From Figure 1c, analysis of the occurrence of individual seismic events within cluster a over time and depth shows no obvious pattern of seismic evolution. If fluid pressure diffusion in a homogeneous medium was the only explanation for the observed seismicity at Açú, one would expect to see events within the cluster migrating uniformly over depth during each year. However, Figure 1c demonstrates that within an individual cluster, events are spatially and temporally scattered. One explanation for this behavior could be the existence of a heterogeneous fault diffusivity structure, which would allow localized pathways of faster and slower flow. These pathways would result in regions of the fault at similar depths varying significantly in the arrival time of the diffusing pressure wave. A further causal factor for this spatial and temporal variability that should be considered is variations in yield strength (the strength threshold for seismic rupture) over the fault plane. We have no direct evidence for variations in fault yield strength, as for the data presented here there is no correlation between earthquake magnitude and either hypocentral depth or time delay. However, we would expect some correlation between yield strength and diffusivity in fractured rock, i.e., areas of the fault that are more heavily fractured may be also weaker. As there is no known consistent relationship between these two properties, we have explored their combined effect through spatial variation of a single property, diffusivity. Therefore the research presented here uses numerical modeling of pressure diffusion through a hydraulically heterogeneous fault to predict the observed temporal and spatial scatter within seismic clusters at Açú, and hence to infer valuable information on the likely internal diffusivity (and possible yield strength) structure of fault planes in this region.

\[
\nabla \cdot (K \cdot \nabla h) = S \frac{\partial h}{\partial t}
\]

(1)

where $K$ is the hydraulic conductivity tensor and $S$ is the specific storage, and $h$ is the piezometric head. In the simple case where $K$ and $S$ are constant, this equation is analogous to the commonly quoted pore pressure-diffusion equation with a diffusivity, $D = K/S$. 

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Figure 1. (a) Earthquake epicenters for events in the Açú area recorded during the three field campaigns (1989, 1990–1991, and 1994–1997). Clusters a, b, and c, referred to in text, are also shown. (b) Daily water depth in the reservoir from 1987 to 1997 in Açú Reservoir. The frequency of events for each month is also shown. White frequency histogram shows the monthly seismic activity as recorded by station BA1 (see Figure 1a), and the black frequency shows the monthly seismic activity occurring in cluster a only. (c) Time delay since the reservoir peak water level of 31 May 1995 for the seismic events in cluster a is plotted against event depth on the left-hand y axis. A corresponding histogram of event frequency for cluster a with a bin width of 8 days is also shown plotted against the right-hand y axis. The thin line is the water depth.
In a previous study of this area described by do Nascimento et al. [2005] we modeled pressure diffusion beneath Açú Reservoir using the above equation assuming that diffusivity in both the fault and rock matrix were spatially and temporally constant. From these simulations, we showed that flow occurred preferentially within a two-dimensional fault plane and could not be modelled using a homogeneous 3-D matrix structure. Further, we also predicted that the maximum pressure change required to trigger seismicity at hypocentral depths was 0.5 kPa, implying that tectonic stress in the faults beneath Açú is close to the critical value for failure.

In reality, it is known that diffusivity is spatially and temporally variable due to two causal factors: variations in effective stress and the spatial heterogeneity of the rock material properties. Examining the first of these factors, variations in effective stress can be due to both increasing confining pressure (generally as a function of depth) and temporal pore pressure changes. In fractured crystalline basement, such as that present at Açú, the effect on the hydraulic conductivity, \( K \), of increasing confining pressure can be modeled by a power law decrease over depth (e.g., Brace et al., 1968; Brace, 1980; de Marsily, 1986; Singhal and Gupta, 1999; Morrow and Lockner, 1994; Saar and Manga, 2004). These studies typically show hydraulic conductivity varying by up to two orders of magnitude from the ground surface to the hypocentral depths at Açú. One recent study has also quantified the effect of confining pressure on specific storage, Song et al. [2004] show that for a single sample of intact granite, specific storage decreases linearly with increasing confining pressure over a range of 7–14 MPa. Above 14 MPa, the effect of increasing confining pressure then rapidly diminishes to give a constant value of \( S \). Assuming tectonic stress in the Açú region increases in proportion to the weight of the overlying rock, the data of Song et al. [2004] show that for depths of greater than 1 km any effect on specific storage is negligible.

As stated earlier, temporal variations in pore pressure also lead to changes in effective stress, resulting in temporal variations in diffusivity; do Nascimento et al. [2005] show that the maximum pore pressure change at 2 km depth beneath Açú Reservoir is 0.5 kPa. On the basis of the data presented by Song et al. [2004] it is clear that such small pressure changes will have a negligible effect, and therefore temporal variations in diffusivity are not considered further within this paper.

The second factor that may be influencing fault diffusivity beneath Açú Reservoir is rock heterogeneity. Field observations of the faults in the Açú region by P. Cowie and A. do Nascimento [do Nascimento, 2002] show them to be typically characterized by zones of intense fracturing that demonstrate both opening and minor shear offsets (Figure 2). This is in agreement with observations of faults in Precambrian basement elsewhere. Dawers and Seeber [1991] called such zones “joint-zones,” and what is significant is that they exhibit very little shear displace-
Figure 3. (a) Regional groundwater flow model for the Açú Reservoir with the reservoir shown in gray. (b) and (c) Refined model with the model fault also shown. The output from sections A, B, C, and D from the regional model were used as lateral boundary conditions for this refined local model. See do Nascimento et al. [2005] for full details of the model configuration.
2. Simulating Pore Pressure Diffusion Beneath Açú

As stated in section 1, diffusion of pore pressure has already been shown to be the predominant mechanism controlling the seismicity in the Açú region [do Nascimento et al., 2005]. To model this process, the groundwater flow code, PARADIGM [Lunn and Mackay, 1997; do Nascimento et al., 2005] has been used. PARADIGM models flow within a 3-D rock matrix with embedded 2-D planar faults. The faults have a separate 2-D planar mesh and can be at any location and orientation; interaction with the 3-D matrix mesh is provided by a fully implicit flow solution that allows leakage between fault nodes and the nearest matrix nodes. Further details of PARADIGM and its application at Açú are presented by do Nascimento et al. [2005].

2.1. Conceptual Flow Model

Figure 3 shows two flow models for the Açú area (Figure 3a), and Figures 3b and 3c show a refined model including the location of a fault plane. The numerical modeling approach is described in detail by do Nascimento et al. [2005]. First, a regional model simulation is performed to provide boundary conditions for the refined model. This regional model is based upon realistic boundary conditions derived from the available climatic and hydrological data in the Açú region [do Nascimento et al., 2005]. Then the refined model, including the fault plane, is run for several years before results are analyzed; this is to remove any spurious effects derived from the initial conditions. The matrix mesh comprises 80 × 80 × 80 elements with an embedded 4 km × 4 km fault plane comprising a further 80 × 80 elements. The north and south boundaries of the model are fixed head boundaries derived from available data and the east, west and bottom boundaries are no flow. A summary of model parameter data is given in Table 1. Appropriate values for specific storage and fault storativity are investigated in detail by do Nascimento et al. [2005] and are taken from this research.

As described by do Nascimento et al. [2005], the surface area of the lake varies temporally within the model simulations since its surface area approximately doubles during the course of a year. The height (or level) of the lake surface is approximated using a sinusoidal oscillation, which is then combined with topographic data to produce the surface area of the reservoir at each time step. The remainder of the upper boundary condition outside the lake (at each time step) is a free surface (the water table). The effect of using a sinusoidal lake variation as opposed to the real lake data is explored by do Nascimento et al. [2005] and it is shown to have only a minor effect on the resulting calibrated diffusivity. This sinusoidal approach has therefore been maintained as it makes interpretation of the pressure diffusion predictions in a heterogeneous medium much clearer.

2.2. Representing Diffusivity Within PARADIGM

As discussed earlier, diffusivity declines with increasing stress and hence depth [Brace et al., 1968; Brace, 1980; de Marsily, 1986; Singhal and Gupta, 1999; Morrow and Lockner, 1994]. This decrease is explained by the general compression and reduction of hydraulic fracture aperture with depth [Rutqvist et al., 1998].


\[
K = K_0 d^{-\alpha}
\]

where \( K_0' = K_0 (\rho g)^{\alpha} \); \( \rho \) and \( g \) are the density of the overburden rock and \( g \) is the acceleration due to gravity; \( K_0 \) is the value of the hydraulic conductivity when no stress is applied; and \( \alpha \) is a constant that measures how the hydraulic conductivity drops off with increasing depth and \( d \) is the depth. On the basis of field or laboratory data on the variation of the hydraulic conductivity with depth, \( K_0 \) and \( \alpha \) are generally found by linear regression [de Marsily, 1986; Lunn and Mackay, 1997; Singhal and Gupta, 1999]. Evans et al. [1997] present a study using basement rocks in which stress-hydraulic conductivity relationships are determined for the damage zone of a fault via laboratory measurements. The value of \( \alpha \) they derive for hydraulic conductivity parallel to the fault plane, perpendicular to the slip direction, as is the case in Açú, is 0.73 for the damage zone.

To incorporate the stress-dependent permeability structure in PARADIGM, equation (2) is used to determine
the values of the input parameter transmissivity, \( T \), over depth in the fault. Fault transmissivity is the hydraulic conductivity integrated over the thickness of the fault plane (required to model two-dimensional flow within the fault). Since \( D = K/S \) this is equivalent to assuming a power law decay in diffusivity, with the same exponent, \( \alpha \). The value of the constant \( K_0 \) in the simulations is found by calibrating the flow model to the observed delay of 4.5 months between the peak in reservoir level and the maximum seismic activity at 2km depth (cluster a, Figure 1b). The value of \( \alpha \) is taken to be 0.73 as determined by Evans et al. [1997]. Varying this exponent within the observed range was found to have a minor effect on model calibration and not to affect the overall conclusions of the paper.

3. Including Heterogeneous Fault Hydraulic Properties in the Groundwater Flow Model

[22] As discussed in section 1, there is substantial field evidence that the transmissivity field in a fault is heterogeneous both because a fault zone is made up by varying degrees of fracturing and hence hydraulic properties, as well as due to the effect of crack closure with increasing depth in the fault zone [Antonellini and Aydin, 1994; Caine et al., 1996; Singhal and Gupta, 1999]. Therefore it is interesting to investigate the diffusion of pressure in fault zones using a heterogeneous transmissivity (and hence diffusivity) field to represent this fracturing in addition to modeling the closure of fractures with increasing depth.

[23] A code based on the turning bands method (TBM) described by Mantoglou and Wilson [1982] has been developed here for the simulation of the spatially correlated fault transmissivity field. Correlated random fields and their generation using TBM have been extensively researched in the simulation of natural hydrologic processes, particularly in groundwater flow and mass transport so the approach taken is well established [Mantoglou and Wilson, 1982; Journel and Huijbregts, 1993; Kitanidis, 1997; Gneiting, 1998]. The underlying properties of the correlated random field are as follows:

[24] 1. The transmissivity fields generated by the TBM code are lognormally distributed, with second-order stationarity.

[25] 2. The structured field is described via the variogram function, \( \gamma(h) \), which is related to the covariance, \( C(h) \), by the Equation \( \gamma(h) = \sigma^2 - C(h) \) as shown in Figure 4a. Here, \( \sigma^2 \) is the a priori variance of the random function of transmissivity and \( h \) is the spatial distance between pairs of points in the random field.

[26] 3. The transmissivity field is defined by its covariance function, range and sill (Figure 4a). Physically, the covariance function describes the relationship between two points a given distance apart, the range is the distance beyond which points are no longer related and just display a background variance of \( \sigma^2 \), and the sill is the value of this background variance, \( \sigma^2 \).

[27] In the TBM the two (or three) dimensional correlated random fields described above are not calculated directly. Instead, the TBM determines values of a derived unidimensional covariance function along several lines. Then at each point of the random field a weighted sum is assigned of the corresponding values on the lines [Mantoglou and Wilson, 1982; Journel and Huijbregts, 1993]. Mantoglou and Wilson [1982] provide a full description of the TBM in two and three dimensions, and it just remains here to select an appropriate covariance function for the faults at Açú and demonstrate that the numerical code developed here does produce realizations of transmissivity with the chosen underlying covariance structure.

[28] If transmissivity, \( T \), field data were available, an experimental variogram could be calculated by plotting the observed covariance of pairs of points a measured distance apart. However, field measurements are not available at Açú and a simple isotropic exponential covariance structure \( C(h) = \sigma^2 e^{-bh} \) has been selected for all the simulations, with a variety of values for the range and sill. The constant \( 1/b \) is termed the correlation length and is equal to one third of the range for an exponential covariance structure. There are several other functions that could have been selected for the covariance, however, all of these basically behave in a similar way (simply varying in shape and steepness as \( h \rightarrow 0 \) so the choice of an exponential function should not greatly influence the overall conclusions. The sensitivity of the pressure simulation results to the values of range and sill will be investigated in the following sections.

[29] Figure 4b (top left) shows an example of a two-dimensional realization of a stationary field generated by the turning bands method. The variance of the field is 0.25 and the correlation length is 167 m. The grid on which the realization is generated has 80 \( \times \) 80 nodes and each node is 50 m apart. Figure 4b (top right) demonstrates the underlying normal distribution (the grey scale is the same as that used to plot the correlated field). Figure 4b (bottom) is the theoretical variogram described by \( \gamma(h) \) (solid line) shown together with an experimental variogram computed from sampling the simulated fields (squares). Figure 4b therefore demonstrates a good match of the simulated transmissivity field to the theoretical statistics. Multiple simulations, with a range of different parameter values, confirmed this result.

[30] To produce the transmissivity fields \( T_f \) to be used in the numerical simulation of the faults at Açú, the following approach was taken to account for the lognormal distribution and the trend in mean hydraulic conductivity produced by the effect of increasing stress over depth (described in section 2.2):

[31] 1. A spatially correlated field \( \alpha_f \) with mean zero (Figure 4b), was produced.

[32] 2. The values of \( \alpha_f \) were inverted to obtain transmissivity values: \( T_{\alpha_f} = 10^{\alpha_f} \).

[33] 3. \( T_{\alpha_f} \) is multiplied by \( T_0 e^{-\alpha} \), where \( T_0 = 3.2 \times 10^{-7} \) m/d and \( \alpha = 0.73 \) (from section 2.2). The result of this final operation is \( T_f \), the transmissivity field.

[34] Figure 5a shows an example of a correlated hydraulic field simulated using this approach with a correlation length of 167 m and a variance, \( \sigma^2 = 0.25 \). Figure 5b shows the mean values of \( T_f \) over depth (calculated from the simulated field in Figure 5a) as triangles connected by a solid black line; the error bars show the standard deviation of \( T_f \) at each depth. The dashed curve is the plot of the transmissivity distribution of the nonuniform fault (\( T_f = 3.2 \times 10^{-2} z^{-0.73} \)) equivalent to the decrease in the mean transmissivity over depth. Figure 5b demonstrates that the reduction in the theoretical mean \( T_f \) with depth is mirrored
Figure 4. (a) Schematic illustration showing some of the properties of both the covariance and the variogram of the transmissivity field [after Journel and Huijbregts, 1993]. (b) Example of two-dimensional realization of stationary fields generated by the turning bands method where \( \sigma^2 = 0.25 \). (top left) A realization with correlation length of 166.67 m. (top right) Histogram of the random field for this realization. (bottom) Analytical (solid lines) and experimental (squares) variograms for the realization.
by the behavior of the experimental mean $T_f$ (the solid black line).

4. Simulation Results of Pore Pressure Diffusion With a Stochastic Transmissivity Field

[35] To investigate the impact of heterogeneity on pore pressure diffusion in the seismogenic fault, simulations with transmissivity fields of different correlation lengths and with different variances have been carried out. The correlation lengths selected for the transmissivity fields are 0 (a random field), 166.67 and 500 m, corresponding to ranges of 0, 500 and 1500 m, respectively. The choice of the largest range investigated is based on the observed size of seismic cluster a shown in Figure 1c: ~1500 m. The lower values of correlation length were chosen to see the impact of increasing correlation length in the numerical simulations.

[36] The fields generated for each simulation (prior to incorporating the effect of increasing stress over depth) have a lognormal distribution with mean zero. To investigate the influence of the variance, two values of 0.25 and 0.5 are investigated. These correspond to standard deviations of 0.5 and 0.71. The fact that the fields have a lognormal distribution, implies that approximately 95.5% of the values of the generated field lie between $-2\sigma$ and $+2\sigma$. Therefore the extreme values of the field are $4\sigma$ apart. The choice of $\sigma^2 = 0.5$ for the simulations is based on the fact that field observations of transmissivity in crystalline rock range from $10^{-3} - 10^{-2}$ m$^2$/d [Rutqvist et al., 1998; Gudmundsson, 2000; Vidstrand, 2001]. In other words, log ($T_f$) in crystalline rocks varies by about 3.

[37] To show the results of the numerical simulations performed for each realization of transmissivity, a plot of the transmissivity field and the piezometric head predictions

![Figure 5](image-url)  
Figure 5. (a) Transmissivity fields with a 167 m correlation length (range 500 m). For this field, $\sigma^2 = 0.25$. (b) Variation of the mean value of the transmissivity at each depth is plotted (dots) together with error bars showing the standard deviation, and the dashed curve shows the theoretical mean transmissivity.

![Figure 6](image-url)  
Figure 6. Demonstration of how the time lag and the amplitude variations are calculated. Only after 226 times steps have been simulated (the dashed line) are the measurements for each node made.
will be shown. Two other outputs of the numerical simulations will also be presented: the corresponding contour plot of the time lag between the maximum water level and the maximum piezometric head prediction for each node of the fault plane and a contour plot of the maximum variation of the piezometric head prediction at each node.

[38] Figure 6 illustrates how the measurements of the time delay and the amplitude variation are made. In order to ensure that all effects of the initial conditions have been removed, 226 time steps (5 years) are first simulated. The lake level and the piezometric head prediction at a node for these 226 time steps are displayed in Figure 6 as dashed lines. When the sixth year begins, the solid lines, the measurements of both the time lag and the amplitude variation are made, for each node of the fault, as illustrated by the double-headed arrows.

Figure 7. Analyzing the time-depth distribution for the earthquakes in cluster a (see Figure 1). Frequency histograms of the percentage of events are shown as a function of the time delay between the peak in water level in October 1995 and the time the earthquake occurs. Each histogram corresponds to a different depth range and the total number of earthquakes is 190. For each depth range, summary statistics are also presented: (a) 1400–1800 m, (b) 1800–2200 m, (c) 2200–2600 m, (d) 2600–3000 m.
4.1. Statistical Description of the Observation Data

In order to make a detailed comparison of the numerical simulation results with the real seismological data, it is necessary to generate some summary statistics from the observation data. A straightforward way to summarize data with a nonparametric distribution is to calculate the range, the median, and the interquartile range (IQR).

Figure 7 shows histograms of the percentage of seismic events at different depths, as a function of the time delay between the peak in water level on 31 May 1995 and the time the earthquakes occur (using data from do Nascimento et al. [2004]). Again, the observation data have been plotted against time from the reservoir peak, since this guarantees that at least the first 270 days of events are caused by pore pressure diffusions, i.e., a drained response. The earthquakes considered here are those from cluster a (Figure 1). Each of the histograms (Figures 7a–7d) show the percentage of the total number of events (190) in a given depth range. The number of events, range, median, and the IQR of the time delay are also shown. Note that no earthquakes were recorded at depths less than 1.5 km. Aseismic fault slip usually occurs also observed. For the deepest depth range considered (Figure 7d), only seven events are available, all occurring at around 136 days. A large scatter in the time lag is also observed. Figure 7b comprises the majority of the seismic events considered. The histogram shows that the peak in the frequency distribution corresponds to a time lag between 128 and 136 days. There is also a large scatter from 8 to 432 days. In Figure 7c the depth range is from 2200 to 2600 m. Here, fewer events are triggered (only 37 events) and a small peak in the frequency distribution is observed at around 136 days. A large scatter in the time lag is also observed. For the deepest depth range considered (Figure 7d), only seven events are available, all occurring at a time lag of zero. A common feature of all the histograms is that there are periods of quiescence of up to 32 days.

Figure 7 shows that in all depth ranges, a few events are recorded within the first 2 month period. It seems very unlikely that these events are triggered by pore pressure diffusion due to the most recent water level peak in the reservoir. They could either be residual effects of pressure diffusion from the maximum reservoir level a year earlier or they could be triggered by an undrained response (seismicity resulting from the instantaneous effect of loading and a delayed pressure diffusion response, see [Talwani, 1997]). Indeed, a combination of the two effects could be present at Açú. Whatever the cause, in the numerical simulations below, only the diffusion effect (i.e., drained response [Talwani, 1997]) is investigated numerically, since it is the dominant mechanism of RIS in the Açú Reservoir and is the only mechanism that provides information on fault hydraulic heterogeneity. Hence, for comparison with these, it is important to calculate the summary statistics from the observation data for events likely to be caused by pore pressure diffusion from the last reservoir level rise only. At 64 days, there is a notable period of quiescence in all of the histograms before the onset of the peak period of seismicity; hence summary statistics have been generated excluding the first 64 days in Table 2.

<table>
<thead>
<tr>
<th>Depth Range</th>
<th>Median</th>
<th>IQR</th>
<th>Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400–1800 m</td>
<td>123</td>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td>1800–2200 m</td>
<td>359</td>
<td>68</td>
<td>120</td>
</tr>
<tr>
<td>2200–2600 m</td>
<td>297</td>
<td>164</td>
<td>32</td>
</tr>
</tbody>
</table>

4.2. Numerical Simulation With a Purely Random Transmissivity Field

The methodology used in this paper is to first investigate a random fault permeability field in which only the effect of crack closure with depth is accounted for and then to add to this field more and more structure, increasing both the correlation length and the variance of the transmissivity field. To ensure consistent initial conditions, for each simulation a full 7 year reservoir cycle is simulated with statistical results calculated using the latter 2 years, following the methodology described by Figure 6.
The time delay to peak pressure change is shown in Figure 8b. The maximum delay is 328 days in the deepest part of the fault. Figure 8c shows the amplitude of the pressure wave (in m), which as expected decays with increasing depth. At 2.0 km depth, this ranges between 0.1 and 0.2 m. For depths greater than 2600 m, the amplitude variation is not larger than 0.05 m, below which no events are observed in this cluster [see Do Nascimento et al., 2005]. The histograms in Figure 8d show the time delays (as defined in Figure 6) at the depth ranges corresponding to the observation data (i.e., Figure 7).

Figure 8. Single realization of a random correlated transmissivity field in the fault plane with a random field and $\sigma^2 = 0.25$: (a) log transmissivity field, (b) time delay, (c) amplitude of the pressure wave, and (d) histogram of predicted events and summary statistics for three depth ranges corresponding to the observation data (i.e., Figure 7).
Table 3. Simulation Summary Statistics for Each Ensemble of Model Simulations for the Three Different Depth Ranges

<table>
<thead>
<tr>
<th>Simulation Type</th>
<th>Depth Range, m</th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1400 – 1800</td>
<td>1800 – 2200</td>
<td>2200 – 2600</td>
<td></td>
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<tr>
<td>Nonuniform</td>
<td>96, 16, 72</td>
<td>128, 16, 72</td>
<td>160, 16, 72</td>
<td></td>
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<tr>
<td>Random field with stress effect, $\sigma = 0.25$</td>
<td>104, 16, 56</td>
<td>136, 16, 56</td>
<td>176, 16, 56</td>
<td></td>
</tr>
<tr>
<td>Random field with stress effect, $\sigma = 0.25$ and stress effect (ensemble statistics)</td>
<td>104, 16, 64</td>
<td>136, 16, 56</td>
<td>176, 16, 64</td>
<td></td>
</tr>
<tr>
<td>$b^{-1} = 166.67, \sigma = 0.25$ and stress effect (ensemble statistics)</td>
<td>88, 24, 88</td>
<td>120, 32, 96</td>
<td>160, 32, 120</td>
<td></td>
</tr>
<tr>
<td>$b^{-1} = 166.67, \sigma = 0.50$ and stress effect (ensemble statistics)</td>
<td>88, 32, 112</td>
<td>120, 48, 120</td>
<td>160, 40, 184</td>
<td></td>
</tr>
<tr>
<td>$b^{-1} = 500, \sigma = 0.25$ and stress effect (ensemble statistics)</td>
<td>88, 32, 144</td>
<td>120, 40, 160</td>
<td>152, 40, 184</td>
<td></td>
</tr>
<tr>
<td>$b^{-1} = 500, \sigma = 0.50$ and stress effect (ensemble statistics)</td>
<td>88, 48, 224</td>
<td>120, 48, 224</td>
<td>152, 56, 256</td>
<td></td>
</tr>
</tbody>
</table>

For each depth range and transmissivity field, the three values for each entry are median, interquartile range, and the range.

those calculated from the observation data in Figure 7. The total number of nodes in the whole set of histograms is 1920 and each histogram comprises 640 nodes. The distribution of the percentage of triggered nodes at each depth diverges only slightly from a uniform distribution, which would be anticipated if all the contours were horizontal. This deviation is in part due to the introduction of the random field, but also due to the position of the fault relative to the lake and the regional groundwater gradient; the fault extends well beyond the boundary of the lake at the ground surface; hence not all surface nodes are subject to the sinusoidal head boundary. In general, the median of each histogram increases as depth increases, due to the later arrival of the pressure front at deeper depths; the range and IQR at all depths are much the same since the histogram distributions are near uniform.

It is clear from the poor predictions of range, IQR and median that the process of simulation is not an appropriate model to explain the seismicity pattern. The next stage therefore is to simulate flow through a correlated random field that will provide a network of structured highly permeable and impermeable flow paths.

4.3. Numerical Simulations With Exponentially Correlated Transmissivity Fields

In the simulations presented below, a number of transmissivity fields have been generated for each pair of correlation length and variance following the methodology outlined in section 3. As mentioned before, the aim of this approach is to identify fault transmissivity parameters ($b^{-1}$ and $\sigma^2$ of the transmissivity field) that can reproduce a similar distribution shape and spread (for each depth) as that observed in the frequency histograms of the real data (Figure 7). When dealing with stochastic field simulation it is important to bear in mind that each individual realization of the transmissivity field is a single sample of an infinite number of possible realizations. Hence the values of range, median and IQR for a single realization of the transmissivity field are not statistically significant. The statistics that should be analyzed are those from an ensemble of numerical simulations. Ideally, to produce reliable estimates of population statistics, the ensemble should contain at least 1,000 realizations. However, for the mesh used in this research, the CPU time, for each transmissivity field, is around 8.5h on a SPARC1000. If 20 random realizations were made, one week of CPU time would be necessary for a single set of $b^{-1}$ and $\sigma^2$.

Because of this considerable CPU time, only five runs were made for each set of $b^{-1}$ and $\sigma^2$. Because of this low number of simulations, it is not possible to accurately estimate the exact ensemble summary statistics.

However, it is demonstrated below that we are still able to identify consistent trends in the values of the ensemble statistics, through a sensitivity analysis on the individual parameters of the underlying statistical distributions. To allow direct comparisons of the parameter sets in the following sections, the same five seeds of the random number generator were used for each ensemble of five simulations.

For the first set of spatially correlated simulations, the correlation length chosen is 166.67 m and the variance is 0.25. The summary statistics at each depth range are given in Table 3. With five random realizations of the correlation field some conclusions on the behavior of the median, range and IQR may be drawn. Both the range and IQR have increased at all depths in comparison to those of the uncorrelated field. They also tend to increase in value as the depth of the prediction interval increases. The median of the five realizations, as expected, increases as the depth of the depth interval considered increases. In comparison to the observation data, as with the uncorrelated field, the median is close in value to the observation data below 1800 m, but is much smaller than that of the observation data in the 1400 – 1800 m interval.

Figure 9 shows an example of a single realization from this ensemble of experiment; clusters of contrasting transmissivity are apparent (Figure 9a) resulting in greater heterogeneity within the delay and amplitude plots (Figures 9b and 9c). The histograms for the single realization are shown in Figure 9d. It should be recognized that statistics derived from individual realizations vary significantly and a comparison of the corresponding summary statistics for...
Because of the difference between the statistics derived from a single realization and those of the underlying population, one has to be extremely cautious in inferring statistics from the real data. However, a general comparison is of interest. The values of range and IQR, at the 1800–2200 m depth range, are 96 and 32 days (Table 3). This is
much smaller than the ones found by the real earthquake data: 405 and 68 days. As discussed, the values of correlation length and variance control the range and the IQR. Therefore it is interesting to increase the values of $b^{-1}$ and $\sigma^2$ to further investigate their effect on the results of the ensemble of random realizations considered.

4.3.1. Effect of Independently Increasing Either the Variance or the Correlation Length

Table 3 shows the summary statistics for the five simulations using an increased variance of $\sigma^2 = 0.5$. This produces a corresponding increase in the values of IQR and range for each depth interval. These values,
however, remain well below those derived from the observation data (Table 3). In addition, further analysis also showed very little change in the overall shape of the ensemble histograms.

[54] The effect of increasing the correlation length only to 500 m is now investigated, with the variance maintained at the original value of 0.25. This value for the correlation length has been selected due to the intuitive notion that clusters of high/low transmissivity are associated with the size of the seismicity cloud observed in the real earthquake data beneath the Açú Reservoir [do Nascimento et al., 2005].

[55] Table 3 shows the summary statistics for the ensemble of five random correlated transmissivity fields. Now, a comparison of the summary statistics of this ensemble with the summary statistics in section 4.2 shows that the increase in the correlation length also produced an increase in both the range and IQR. However, again values are well below those derived from the observation data.

4.3.2. Effect of Increasing Both the Correlation Length and the Variance

[56] In this section, simulations are investigated with a value of 500 m for the correlation length and a variance of 0.5. Table 3 shows the summary statistics for the three depth intervals.

[57] With \( b^{-1} = 500 \) m, the increase in \( \sigma^2 \) from 0.25 to 0.50 has produced a great increase in the range of the ensemble of triggered nodes, due to the sensitivity of this statistic to extreme values. By comparison, the IQR is less sensitive to extremes, nevertheless, it has also increased significantly in comparison to previous simulations (Table 3). For the ensemble considered, the values of range and IQR, at the 1800–2200 m depth range, are 224 and 48 days, which is still smaller than the ones derived from the real earthquake data: 359 and 68 days. However, inspection of the individual simulations shows a wide range in sample statistics even though there are only five realizations. In fact, between realizations the IQR varies from 28 and 56 days at 1800–2200 m depth and from 32 and 88 at 2200–2600 m depth. Since the real world is a single realization, it seems entirely possible that the observed values could be sampled from an underlying population with similar statistics given a sufficiently large number of realizations.

[58] Figure 10a shows a single realization of the random correlated field with \( b^{-1} = 500 \) m and \( \sigma^2 = 0.50 \). The increase in variance has produced a noticeable change in the behavior of the delay plot. “Pockets” are apparent in all five model time-delay plots. For example, the presence of such a “pocket” is indicated on Figure 10b. These occur because the regions of low transmissivity seen in Figure 10a are extensive enough to impede the flow. The diameter of each pocket is approximately equal to the diameter of the corresponding cluster of low transmissivity. These pockets lead to a bimodal distribution in the histograms for individual realizations. It is interesting to note that similar bimodal behavior is found in the observation data in Figure 7b for the 1800–2200 m depth interval.

5. Discussion and Conclusions

[59] The heterogeneous nature of fault hydraulic properties at Açú has been demonstrated by local field observations and a literature review of field and laboratory studies of fault diffusivity from other locations. Evidence of such heterogeneity is also apparent from analysis of seismicity data in the Açú region [do Nascimento et al., 2004]. Continuous digital seismic monitoring revealed that seismicity occurs in recognizable clusters, the location and activation of which are consistent with a causal mechanism of pore pressure diffusion. Within seismic clusters, however, the events are observed to be spatially and temporally scattered. We show here that these observations only remain consistent with pore pressure diffusion if the fault is hydraulically (and structurally) heterogeneous with a high degree of spatial correlation and a large variance in its transmissivity structure. As discussed in the introduction, we expect diffusivity and rupture yield strength to be spatially correlated, and hence simulation results represent their combined effect.

[60] To produce hydraulically realistic faults, the diffusivity structure has been modelled here using a lognormally distributed random transmissivity field with an exponential covariance structure. The effect of increasing normal stress with depth has then been incorporated by applying an exponentially reducing trend in the mean transmissivity over depth. This hydraulic modeling approach is confirmed by field and laboratory data from the literature.

[61] To compare the observed seismicity data with the numerical simulations of pore pressure diffusion, summary statistics were derived from the observation data for seismic cluster a (Figure 1). Monte Carlo simulations of pore pressure diffusion were performed using five realizations of each transmissivity model. These simulations summarized in Table 3 showed that as the variance and correlation length were increased to values of 500 m and 0.5, respectively, the summary statistics approached those of the observation data. So, despite the high variance in the transmissivity field, it is the high degree of spatial structure that is crucial in explaining the observation data. It is interesting to highlight a number of characteristics from the results of this highly structured transmissivity model. First, a correlation length of 500 m (equivalent to a range of 1500 m) confirms the intuitive notion that the clusters of high and low transmissivity may be associated with the size of the seismic cloud observed in the real earthquake data. Second, a variance of 0.5 implies that 95% of the transmissivity values lie within approximately 3 orders of magnitude at any given depth. This large range of values supports the high variation in field measurements documented in the literature. It may even be the case that the real variance could be further increased, since field experiments are derived from pump test data, which necessarily produce an average transmissivity value for a given volume of rock. Third, the range and interquartile range of our simulated results remain smaller than those from the data when ensemble statistic are calculated. However, analysis of individual realizations shows great variation in these properties and it is entirely possible that the real world, which is itself a single realization could be described by similar population statistics. Finally, analysis of individual realizations shows a tendency to produce pockets of high and low transmissivity in the fault plane which result in bimodal histograms of seismic triggering. This mimics the behavior found at 1800–2200 m depth in the observation data.
The numerical simulations presented here have demonstrated that the spatiotemporal evolution of seismicity within clusters at Açú can be explained by pressure diffusion through a fault with a spatially correlated transmissivity (and hence diffusivity) field. Further, these detailed observation data have added new insights as to the likely structure of that field and provided estimates of the underlying correlation length and variance that are currently not possible to derive by any other means.

Acknowledgments. A.F.D.N. was supported by a scholarship from the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brazil). A.F.D.N. thanks the projects “Caracterização geomecânica de reservatórios heterogêneos” and “Falhas e fraturas naturais: aplicações na caracterização de reservatórios” from FINEP/CTPETRO, and the Agência Nacional do Petróleo—ANP, Brazil for financial support. GMT [Wessel and Smith, 1998] was used to display most of our results. PC was supported by the Royal Society. We thank two anonymous reviewers for constructive comments on this manuscript. Most of the figures were prepared using GMT [Wessel and Smith, 1998].

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