A Technique for the Aperture Partitioning

Amedeo Capozzoli, Claudio Cercio, Giuseppe D’Elia, Angelo Liseno, Francesco Marano
Università di Napoli Federico II
Dipartimento di Ingegneria Elettrica e delle Tecnologie dell’Informazione (DIETI)
Via Claudio 21, I-80125 Napoli, Italia
a.capozzoli@unina.it

Abstract—This paper presents a method for the partitioning of an aperture into sub-apertures preserving the performance. The technique represents a first stage towards a subarray synthesis and is based on a multi-stage approach wherein an aperture synthesis, an evolutionary algorithm, and, finally, a local search are adopted. A proper representation has been considered to reduce the number of the parameters defining the sub-aperture partitioning and simplify the use of a global optimizer.

Keywords—subarray, sub-aperture, synthesis, global optimizer.

I. INTRODUCTION

In many applications there is the need for electronically steerable or reconfigurable beam antennas. Typically a phase-only beam control is required. Anyway, in spite of this simplification, such antenna systems require a complex and quite expensive beamforming network (BFN). The use of subarray technology, as demonstrated by the limited-field-of-view systems, is useful to reduce the costs, by keeping low the number of control components. Accordingly, a proper strategy should be adopted to reduce the effects of the undesired lobes due to the use of a subarray arrangement, with attention to the subarrays shapes, number and distribution.

Different solutions have been proposed in the literature. The method of the overlapped subarray network has been proposed in [2]. It is suited for very limited scan range and very narrow bandwidth, even if it guarantees a relevant hardware saving (phase-shifters or time delay devices) [2]. On the other hand the use of irregular subarrays have been also proposed [3,4] and in the references therein contained. Obviously, designing arrays with these features requires subarray synthesis approaches, i.e. techniques able to determine the partitioning of an array into subarrays and the excitation of the elements, whose control phases are assumed uniform across each subarray.

Under this point of view, the use of subarrays with irregular sizes and/or shapes, such as the Polyomino-shaped subarrays [4,5], properly arranged, represents an interesting solution to reduce the undesired lobes in the radiation pattern. Promising results have been obtained by using genetic algorithms in [6].

The aim of this paper is to propose an approach to the Sub-Apertures Synthesis (SAS), i.e. to partition an aperture antenna by preserving its performance. This method should be intended as a first step of a subarray synthesis. In fact, the study of the aperture partitioning, at the beginning, allows to deal with a continuous problem, with a reduction of the number of unknowns, and the generation of a good starting point useful in the subsequent design step wherein the discrete nature of the antenna (array) is explicitly taken into account. More in detail, for a given array aperture, the first step of the synthesis can be considered as the one segmenting the aperture into sub-apertures, with proper shapes and phases, so that the radiated far field pattern is as "close" as possible to the one corresponding to the optimal (unsegmented) aperture field distribution.

In principle, a successful SAS could operate by subdividing the aperture into polygonal, non-overlapping, sub-apertures wherein the phase is assumed constant, and by finding the shapes of the sub-apertures and the related phases, guaranteeing the coverage of the whole aperture. However, by taking into account that a generic polygon can be always partitioned into triangles, the SAS can always deal with triangles instead of polygons, thus simplifying the task. Indeed, a triangle is completely defined by its vertices, and so, by acting on an irregular grid of nodes (vertices) covering the aperture, it is possible to generate the desired aperture partitioning.

In the following the optimization of the non uniform grid is performed by means of an evolutionary algorithm, while that of the sub-apertures phases by a local search of the quasi-Newton class. It’s worth noting that acting on the grid of vertices makes the problem continuous instead of combinatorial. Furthermore, a proper representation of the unknowns defining the grid allows to control the number of the unknowns (even with a drastic reduction), with beneficial effects on the optimization procedure [7]. This advantage can be further enhanced by a progressive enlargement of the unknowns.

The key ideas here proposed are the following:

- To find the most convenient sub-aperture partitioning and phases to reduce the detrimental effects of the subarray partitioning on the far field pattern;
- To partition the antenna aperture into triangular sub-apertures, which allow flexibility, guarantee a covering without gaps or overlapping, and allows to turn a
combinatorial optimization into a continuous optimization;
- To control the shapes and the locations of the triangles by acting on the grid of their vertexes;
- To think the unknown grid of the vertexes of the triangles as the results of the distortion of a known regular grid via an unknown mapping;
- To expand the unknown mapping on a set of known basis function and assume the expansion coefficients as the actual unknowns of the aperture partitioning problem (effective control of the number of real unknowns);
- To exploit a mixed deterministic and stochastic (evolutionary) optimizer to find a satisfactory trade-off between efficiency and effectiveness.

II. THE APPROACH

Let us refer to an aperture $\mathcal{A}$, $2a_{\text{ap}} \times 2b_{\text{ap}}$ sized (see Fig.1), associated to a given planar array, and let us imagine that an aperture synthesis has been performed, so that the aperture field $E_a$ is known. This is the first step of the SAS. For the sake of simplicity let us refer to a linearly polarized aperture field, $E_a = E_a \hat{x}$.

Furthermore, let us assume that a grid of vertices (nodes) covering $\mathcal{A}$ has been introduced, defining the partitioning of the aperture into triangles. In the illustrative picture in Fig. 2, the nodes of one sub-aperture are highlighted as green circles. It can be noted that, in this case, the nodes are uniformly distributed over $\mathcal{A}$. As a starting phase of each sub-aperture we will assume the mean value of the phase of the aperture field $E_a$ synthesized at the beginning.

By referring to Fig. 4, the first design step is an aperture synthesis, aimed to find the aperture field minimizing a given objective functional. In particular, we considered a power pattern synthesis wherein the pattern specifications are given in terms of mask functions [8]. Several approaches can be adopted. The results here have been obtained by using the alternate projection method [8], wherein the aperture field is expanded by means of Prolate Spheroidal Wave Functions (PSWF) [9], useful to get a very convenient representation of the aperture field [10]. More in detail $E_a$ can be written as:

$$E_a(x, z) = \sum_{p=1}^{P} \sum_{q=1}^{Q} \alpha_{pq} \Phi_p \left[ c_x, x \right] \Phi_q \left[ c_z, y \right]$$

where $\Phi_i(c_w, w)$ is the $i$-th, 1D PSWF with “space-bandwidth product” $c_w$ [11], $c_x = a_{\text{ap}} \beta$, $c_z = b_{\text{ap}} \beta$, $\beta = 2\pi/\lambda$, $\lambda$ being the wavelength, $P = \text{Int}[4a/\lambda]$ and $Q = \text{Int}[4b/\lambda]$, $\text{Int}[\cdot]$ denoting the integer part of its argument.
Fig. 2. Example of an aperture subdivided into uniform triangular sub-apertures: the nodes of one subaperture are highlighted as green circles, while \( \mathcal{A} \) is delimited by the purple line.

Fig. 3. Example of an aperture subdivided into non uniform triangular sub-apertures: the positions of half-wavelength spaced array elements filling the given aperture are highlighted as green crosses, while \( \mathcal{A} \) is delimited by the purple line.

The objective functional considered in this paper is given by:

\[
\Phi (\alpha) = \|F(\alpha)\|^2 - P_\mathcal{Y} \left( \|F(\alpha)\|^2 \right) \|
\]

where \( \alpha \) is the vector containing the expansion coefficients of \( E_a \), \( F \) is the copolar far field pattern, \( P_\mathcal{Y} \) is the projector onto the set \( \mathcal{Y} \) containing all the power patterns satisfying the design specifications, and \( \| \cdot \| \) is a properly weighted norm, with weight \( v \).

The selected aperture synthesis is quite general and employable either in the case of shaped and pencil beam patterns as shown in [9].

### B. The functional optimization and the unknowns representation

Once the aperture field is known, the aperture is divided into sub-apertures and the positions of the sub-aperture nodes are found by means of an evolutionary algorithm [7,12]. Obviously, the direct optimization of the nodes coordinates, by using a global tool, can be very cumbersome when the number of sub-apertures is large, as in the case of wide apertures. To this end, a proper representation of the unknowns can be adopted, as in [9,13]. The basic idea is that the non uniform nodes distributions can be obtained starting from a regular grid thanks to a "distortion function", expanded by means of basis functions. This strategy controls the number of parameters to be optimized, which turns into the expansion coefficients, with even a drastic reduction.

More in detail, the non uniform grid, say \( (x^{(m)},y^{(m)}) \), is obtained by distorting a regular auxiliary grid \( (\xi^{(m)},\eta^{(m)}) \), via the mapping function \( w \), to be determined:

\[
(x^{(m)},y^{(m)}) = w(\xi^{(m)},\eta^{(m)})
\]

The mapping function \( w \) is represented by means of \( W \) basis functions (e.g., polynomials), say \( \tau \):

\[
w(\xi^{(m)},\eta^{(m)}) = \sum_{s=1}^{W} g_s \tau_s(\xi^{(m)},\eta^{(m)})
\]

where the coefficients \( g_s \) are the new unknowns of the problem.

Finally, once the optimal sub-apertures partitioning has been found, the phase of the sub-apertures is refined, by means of a local optimizer. This final step allows a further improvement on the radiated pattern with respect to the results corresponding to the previously considered mean phase values.

### III. RESULTS

The case here reported refers to a square aperture antenna \( 10\times10 \) sized. The antenna is required to radiate a pencil beam along \( u_0=0.34 \) and \( v_0=0.34 \), where \( u_0 \) and \( v_0 \) are the cosine directors associated to the considered beam pointing angle.

The pattern (copolar component) obtained from the aperture synthesis is reported in Fig. 5. Its maximum directivity is equal to 29.5dB.

Before showing the results of the nodes optimization, let us consider the case of a uniform node distribution. To this end, let us refer to an effective aperture \( \mathcal{A}' = 2a'_a \times 2b'_a \) sized,
wherein $a'_n=\gamma a_n$ and $b'_n=\gamma b_n$, and let us divide $A_{\text{eff}}$ into $T$ triangular, uniformly distributed, sub-apertures (see Fig. 6), with $\gamma=1.5$ and $T=200$ in the case of interest here. The use of an enlarged $A_{\text{eff}}$ is useful to avoid a constrained optimization of the nodes. Indeed, in this way the nodes can be freely moved without creating "holes" in the sub-apertures subdivision.

The corresponding radiated pattern is reported in Fig. 7 (obviously only the triangles within $A$ are excited with the synthesized aperture field, the phase being kept constant over each sub-aperture, as specified before). As seen, the pattern is seriously degraded, and the maximum directivity (24.6dB) falls down of about 5dB with respect to the continuous aperture case. In Figs. 8-9 the cuts along $u=u_0$ and $v=v_0$ are shown, respectively, together with the mask functions defining the pattern specifications.

The optimization of the partitioning with the evolutionary algorithm has been performed by using a population of 150 individuals and a maximum number of 200 generations. Each individual is made up by 19 unknowns: 18 coefficients are adopted to represent the nodes coordinates through the mapping function, wherein Legendre polynomials of the 3rd order are considered, and one more unknown has been introduced to allow a rotation of the whole grid in the aperture plane.

The optimized sub-apertures partitioning is shown in Fig. 10. The number of sub-apertures falling within $A$ is equal to 147, with 100 nodes. Accordingly, the number of the unknowns exploited is drastically smaller than the number of unknowns which should be employed without the mapping function $w$ (2x100) and drastically smaller than the number of triangles. The elements of an half-wavelength spaced array filling $A$ are also highlighted in Fig. 10 as green crosses. This shows how the array elements would be grouped together according to the proposed partitioning scheme. It’s worth noting that the minimum tolerable sub-aperture size can be controlled by inserting a constraint on the minimum distance among the nodes of the non uniform grid.

The final synthesized pattern, achieved after the sub-aperture phase refinement, is shown in Fig. 11. The corresponding maximum directivity is equal to 28.8dB, quite close to that of the aperture synthesis. In Figs. 12-13 the radiated pattern along $u=u_0$ and $v=v_0$ are shown, respectively.

The synthesized phase, constant over the sub-apertures, is shown in Fig 14.

IV. CONCLUSIONS

An approach to the aperture partitioning synthesis has been presented. The approach turns the combinatorial problem of filling the aperture of the antenna with sub-apertures of different shapes without gaps and overlapping, a sort of knapsack problem, into a continuous problem with a number of unknowns that can be easily controlled thanks to a proper representation. The numerical analysis shows that, to reduce the degradation of the far field pattern, due to the aperture partitioning, and get a far field pattern close to the nominal one of a continuous aperture, the number of parameters defining the partitioning can be drastically smaller than that of the sub-apertures, with beneficial effects on the optimization process. The effectiveness and the efficiency of the optimization is achieved by using a mixed solution based on an evolutionary algorithm, managing the partitioning, and a deterministic one, searching for the phases of each sub-aperture.
Fig. 7. Pattern corresponding to a uniform sub-aperture distribution

Fig. 8. Amplitude of the far field pattern along the cut $u = u_0$ (red line) together with the mask functions (blue line): uniform sub-aperture distribution

Fig. 9. Amplitude of the far field pattern along the cut $v = v_0$ (red line) together with the mask functions (blue line): uniform sub-aperture distribution

Fig. 10. Optimized subapertures distribution: the positions of half-wavelength spaced array element corresponding to the given aperture are represented as green crosses, while $\mathcal{A}$ is delimited by the purple line

Fig. 11. Synthesized pattern: optimized subapertures distribution

Fig. 12. Amplitude of the far field pattern along the cut $u = u_0$ (red line) together with the mask functions (blue line): uniform subaperture distribution
Fig. 13. Amplitude of the far field pattern along the cut $v=v_0$ (red line) together with the mask functions (blue line): uniform subaperture distribution

Fig. 14. Synthesized sub-apertures control phase

ACKNOWLEDGMENT
The Authors wish to acknowledge Dr. Giovanni Toso for the discussions on the topic.

REFERENCES