Application of fuzzy set theory to evaluate the probability of failure in rock slopes

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ABSTRACT

Because uncertainty pervades the field of rock slope stability analysis, the importance of uncertainty has been recognized. Subsequently, probability theory has been used to quantify the uncertainty. However, some uncertainties, due to incomplete information, cannot be managed satisfactorily by probability theory, so fuzzy set theory is more appropriate in the case. In this study, the uncertain parameters in rock slope stability analysis were expressed as fuzzy numbers and fuzzy set theory was employed. The Monte Carlo simulation technique and reliability index approach were implemented with fuzzy set theory in order to take into account the fuzzy uncertainties in the evaluation of the probability of failure. In order to check the feasibility of the proposed approaches, the presented methods were applied to a practical example. Based on the results of the practical application, it was concluded that the application of fuzzy set theory shows consistent analysis results and can obtain reasonable results.

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1. Introduction

One of the difficulties in slope stability analysis is the inevitable uncertainty involved in the material properties and the geotechnical model. The natural materials comprising most slopes have an innate variability that is difficult to establish and predict; therefore, the variability of the geologic material is a major source of uncertainty. An insufficient amount of information about site conditions and an incomplete understanding of the failure mechanism are other sources of uncertainties. Therefore, the presence and significance of uncertainties in slope stability analysis have long been appreciated. Consequently, several approaches, such as the observation method (Peck, 1969), have been suggested to deal properly with uncertainty.

The probabilistic approach has also been used as a powerful tool for representing uncertainty in the failure model and in the material characteristics. Many probabilistic analyses have been published in the literature (Einstein and Baecher, 1982; Mostyn and Small, 1987; Mostyn and Li, 1993; Nilsen, 2000; Park and West, 2001; El-Ramly et al., 2002; Pathak and Nilsen, 2004; Park et al., 2005). However, a large amount of data is important to accurately implement probabilistic analysis (Haldar and Mahadevan, 2000; Giasi et al., 2003). In other words, it is more desirable to use an adequate number of reliable observations to estimate uncertainty. However, in practical conditions, the amount of data is frequently limited, and the distribution type of the uncertain variable may not be known. This situation makes the application of the probabilistic approach difficult (Dodagoudar and Venkatachal, 2000).

Furthermore, some uncertainties connected to measured geotechnical parameters may, in fact, be non-stochastic in nature (Juang et al., 1998; Nawari and Liang, 2000). That is, not all uncertainties are random or objectively quantifiable. Some uncertainties, especially those based on incomplete information, are due to cognitive sources (Zimmermann, 2001). Under such conditions of limited information, it appears reasonable to base estimations on the concepts of fuzzy set (Dodagoudar and Venkatachal, 2000).

Fuzzy set theory was proposed by Zadeh (1965). It has been recognized as an appropriate approach for dealing with fuzzy uncertainty that is mainly caused by incomplete information; therefore, fuzzy set theory has been used for geological engineering and rock mechanics problems (Ghose and Dutta, 1987; Grima and Babuska, 1999; Yao et al., 1999; Finol et al., 2001; Gokceoglu, 2002; Kayabasi et al., 2003; Gokceoglu and Zorlu, 2004; Chen and Liu, 2007; Hamidi et al., 2010). Especially, fuzzy set theory has been employed in several slope stability analyses (Juang and Lee, 1992; Lee and Juang, 1992; Davis and Keller, 1997; Juang et al., 1998; Dodagoudar and Venkatachal, 2000; Giasi et al., 2003; Li and Mei, 2004). The previous studies used fuzzy set theory with the vertex method and an approximate method, such as the point estimate method or the first-order second-moment method. Consequently, these previous analyses could not provide accurate analysis results even if the fuzzy approach is computationally simple and robust to changes in the shape of...
input distributions (Zonouz and Miremadi, 2006). On the other hand, the Monte Carlo simulation, one of the most commonly used probabilistic analyses is the most complete and accurate method and subsequently a lot of slope stability analyses utilized this approach (Nilsen, 2000; Park and West, 2001; El-Ramly et al., 2002; Zhou et al., 2003; Pathak and Nilsen, 2004; Xie et al., 2004; Park et al., 2005; Liu, 2008; Wang et al., 2008, 2010; Shou et al., 2009). However, Monte Carlo simulation is sensitive to uncertainty about input distribution and needed to assume correlation among all inputs. Nevertheless, the fuzzy theory and Monte Carlo simulation, and their hybrid approaches have increasingly been used in reliability analysis since both of fuzzy set theory and Monte Carlo simulation have successfully dealt with uncertainties.

This study proposes two hybrid fuzzy techniques which incorporate Monte Carlo simulation and the reliability index approach in order to take fuzzy uncertainty into account during reliability analysis of slopes. The fuzzy based Monte Carlo simulation utilized the fuzzy number concept and Monte Carlo simulation simultaneously in order to analyze the stability of a rock slope whereas the fuzzy based reliability index approach combined fuzzy number arithmetic and reliability index concept. To verify their feasibility and validity, the proposed approaches are applied to a practical example.

2. Fuzzy set theory

2.1. Fuzzy set theory and alpha cut level

The fuzzy set theory, introduced by Zadeh (1965), facilitates analysis of non-discrete natural process or phenomenon (Zimmermann, 2001). While the classical set theory defines an object as a member of a set if it has a membership value of 1 or is not a member if it has membership value of 0, the membership of a fuzzy set is expressed on a continuous scale from 1 (full membership) to 0 (full non-membership) (Regmi et al., 2010). In fuzzy set theory, an element's membership function may admit some uncertainty, so membership is expressed as a degree of belonging to a set. The membership function can be manifested by many different types of functions and by the different shapes of their graphs. Triangular and trapezoidal shapes are the most common types of membership function.

The $\alpha$-cut of a fuzzy set $A$ is the crisp set comprised of all the elements $x$ of a universe of discourse $X$, for which the membership function of $A$ is greater than or equal to $\alpha$. The basic idea of the $\alpha$-cut concept is the discretization of a fuzzy number into a group of $\alpha$-cut intervals. For each of the uncertain parameters, the $\alpha$-cut of a fuzzy set will give an interval having two points, i.e. upper and lower bound values for a particular $\alpha$-cut (Figure 1). Any particular fuzzy set $A$ can be transformed into an infinite number of $\alpha$-cut sets, because there are an infinite number of values $\alpha$ on the interval $[0, 1]$.

2.2. Fuzzy number and fuzzy arithmetic operation

There are standard and normal fuzzy sets defined on the set of real numbers, whose $\alpha$-cuts are closed intervals of real numbers and whose supports are bounded. Any fuzzy interval $A$ for which $A(x) = 1$ for exactly one $x$ is called a fuzzy number (Demico and Klijn, 2004). Every fuzzy interval $A$ may be expressed in the general form:

$$A(x) = \begin{cases} f(x) & \text{for } x \in [a, b] \\ 1 & \text{for } x \in [b, c] \\ g(x) & \text{for } x \in [c, d] \\ 0 & \text{otherwise} \end{cases}$$

where $a \leq b \leq c \leq d$, $f(x)$ is a continuous function that increases to 1 at point $b$, and $g(x)$ is a continuous function that decreases from 1 at point $c$.

Given two closed intervals of real numbers, the result of any arithmetic operations on these intervals is defined as the set of real numbers obtained by performing the operation on each ordered pair of real numbers (Cai, 1996).

If $A = [a_1, a_2, a_3]$ and $B = [b_1, b_2, b_3]$ are both triangular fuzzy numbers, then it can be shown that $A + B$ and $A - B$ are also both triangular fuzzy numbers (Kaufmann and Gupta, 1991; Cai, 1996), and evaluated as:

$$A + B = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$$

and

$$A - B = [a_1 - b_3, a_2 - b_2, a_3 - b_1]$$

If $A = [a_1, a_2, a_3, a_4]$ and $B = [b_1, b_2, b_3, b_4]$ are both trapezoidal fuzzy numbers, then it can be shown that $A + B$ and $A - B$ are both trapezoidal fuzzy numbers (Kaufmann and Gupta, 1991; Cai, 1996), and expressed as:

$$A + B = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4]$$

and

$$A - B = [a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1]$$

3. Fuzzy based Monte Carlo simulation

Probabilistic analysis has been the most formidable tool for dealing with uncertainty. However, the probabilistic characteristics of a random variable would be described completely if the form of the distribution function and the associate parameters are specified (Dodagoudar and Venkatachalam, 2000). In addition, as Harrison and Hudson (2010) pointed out, probabilistic analysis is carried out on the premise that, for an uncertain parameter, the precise mean, standard deviation, and an appropriate probability density function can be obtained. However, in some practices, the form of the distribution function may not be known. Frequently, only the maximum and minimum values for an uncertain parameter can be obtained precisely; therefore, an uncertain parameter can be expressed only with an interval between the minimum and the maximum. Under this condition, an uncertain parameter may be expressed as a fuzzy number, if there are some reasons to believe that not all values in the interval have the same degree of support (Juang et al., 1998).

Since fuzzy uncertainties are common in the procedure of slope stability analysis, several research works have deployed fuzzy set theory in slope stability analysis (Juang et al., 1998; Dodagoudar and Venkatachalam, 2000; Giassi et al., 2003). The previous studies
utilized the vertex method (Dong and Wong, 1987) to obtain fuzzy input parameters and analyze the stability of the slope. The vertex method is based on the $\alpha$-cut concept of fuzzy numbers and involves an interval analysis. The basic idea of the vertex method is the discretization of a fuzzy number into a group of $\alpha$-cut intervals. By replacing fuzzy numbers in the slope model with the intervals obtained from the vertex method, the factors of safety are obtained by fuzzy computation. However, when the reliability of a slope is evaluated using the vertex method in the deterministic slope model, an approximate method, such as the first-order second-moment method (Giasi et al., 2003) or the point estimate method (Dodagoudar and Venkatachalam, 2000), has been applied. These methods employ a simple calculation that only requires a few representative values of the uncertain variable without distribution information in order to evaluate the probability of failure (Harr, 1987). However, the previous research studies used the approximate method, approximate analysis results were obtained, instead of precise analysis results.

Therefore, this study proposes the approach that utilizes the fuzzy number concept and Monte Carlo simulation in order to analyze the stability of a rock slope. For reliability analysis, Monte Carlo simulation is recognized as being a more complete and accurate method than the approximate methods. The advantages of Monte Carlo simulation are that it is relatively easy to implement on a computer and it can deal with a wide range of functions, including those that cannot be expressed conveniently in an explicit form (Baecher and Christian, 2003). Therefore, several researches suggested fuzzy based Monte Carlo simulation approach in many different engineering fields (Wonnerberger et al., 1995; Guyonnet et al., 2003; Zonouz and Miremadi, 2006; Buckley and Jowers, 2008; Sadeghi et al., 2010). The main idea of the fuzzy based Monte Carlo simulation (or Fuzzy Monte Carlo Simulation) is to illustrate how to generate random fuzzy number by a Monte Carlo simulation approach. This method covers useful properties of both fuzzy arithmetic and Monte Carlo simulation techniques which are uncertainty propagation and independence of exact mathematical equation of the whole system (Zonouz and Miremadi, 2006). The Fuzzy Monte Carlo simulation approach includes two important phases: fuzzy number generation and simulation stages. However, the previous researches made many different attempts in fuzzy number generation and simulation procedures. Wonnerberger et al. (1995) performed fuzzy to random transformation in random number generation before performing the simulation but there is no fully accepted way of transforming one to another (Pedrycz and Gomide, 1998). Guyonnet et al. (2003) proposed hybrid approach considering Infimum (Inf) and Supremum (Sup) values of the $\alpha$ cuts aggregation and Monte Carlo simulation simultaneously. But in this case only the extreme case of the input value was considered and the analysis results were not similar to the traditional Monte Carlo simulation approach. Meanwhile, the fuzzy number generation using fuzzy arithmetic and Inverse Transform method was proposed by Zonouz and Miremadi (2006). Later Buckley and Jowers (2008) suggested the novel procedure which produces random sequences of fuzzy numbers and fuzzy vector, and then the values were used in Fuzzy Monte Carlo simulation. Recently, Sadeghi et al. (2010) proposed unique Fuzzy Monte Carlo simulation framework that construct a fuzzy cumulative distribution function (CDF) to represent uncertainty.

In this research, however, we present the new hybrid approach using Monte Carlo simulation jointly with the fuzzy random interval of $\alpha$ cut values in random number generation procedure. In order to deploy fuzzy based Monte Carlo simulation in this study, an uncertain parameter is considered to be a fuzzy number and its membership function is decided by means of available information and engineering judgment. Monte Carlo simulation is then carried out with the fuzzy numbers. In the simulation procedure, $\alpha$-level values are randomly selected between 0 and 1 using a random number generator and subsequently, the interval between two elements (the upper and lower $x$ values of an $\alpha$-cut) is obtained from an $\alpha$-cut operation using the randomly generated $\alpha$ values (Figure 2). Then, another random number generation is used in order to randomly select the element values between the upper and lower bound values for each iteration, and these values are used as input parameter values to evaluate the factor of safety for the slope. After repeating this process many times, a large number of different factors of safety are obtained, and the probability of failure can be evaluated. This novel approach proposes the unique hybrid procedure of fuzzy number generation to reduce computational effort and the complex mathematical calculations performed in the previous approaches.

4. Fuzzy reliability analysis

4.1. Concept of probabilistic reliability theory

In order to analyze the stability of a slope, the capacity (or resistance) and demand (or load) on the slope should be evaluated and compared. Slope failure occurs when the demand exceeds the capacity; therefore, the slope is unstable when the factor of safety is less than 1 (since the factor of safety is the ratio of capacity over demand). However, reliability theory uses the concept of a safety margin (SM), which is the difference between the capacity and the demand. Thus, the slope is unstable when SM is less than 0. In a reliability analysis, uncertainty dictates that both the capacity and demand can take on a wide range of values, so both the capacity and demand are considered random variables. Consequently, the safety margin should also be considered as a random variable, because the difference between two random variables is also a random variable.

If both the capacity ($C$) and demand ($D$) are considered to be normally distributed, then the safety margin is also normally distributed, which means that the probability of failure for a slope can be evaluated from a simple closed-form solution. In this condition, the mean value ($\mu_{SM}$) and standard deviation ($\sigma_{SM}$) of safety margin are obtained as:

$$\mu_{SM} = \mu_C - \mu_D$$

$$\sigma_{SM}^2 = \sigma_C^2 + \sigma_D^2$$

Fig. 2. Concept of fuzzy Monte Carlo simulation.
where \( \mu_c \) and \( \mu_d \) are the mean values of the capacity and demand, respectively; and, \( \sigma_c \) and \( \sigma_d \) are the standard deviation of the capacity and demand, respectively. Since the probability distribution of the safety margin (SM) is known to be a normal distribution, the probability of failure can be calculated as:

\[
P_f = P(C < D) = P(C - D < 0) = P(SM < 0).
\]

(8)

4.2. Fuzzy based reliability index

In this approach, capacity and demand can be considered as fuzzy numbers. For simple fuzzy operations, a fuzzy number can be expressed as an interval or sometimes as an alpha-cut interval. Therefore, if capacity and demand are designated as fuzzy numbers, they can be defined as intervals:

\[ C = [c_1, c_3] \]  
\[ D = [d_1, d_3] \]  

(9)  
(10)

If the fuzzy numbers are expressed by an \( \alpha \)-cut interval, the intervals are:

\[ C_\alpha = [c_1(\alpha), c_3(\alpha)] \]  
\[ D_\alpha = [d_1(\alpha), d_3(\alpha)] \]

(11)  
(12)

As mentioned previously, safety margin \( Z \) is the difference between capacity and demand \((C - D)\), and \( Z \) is a fuzzy number because \( C \) and \( D \) are fuzzy numbers. Using fuzzy arithmetic, as in Kaufmann and Gupta (1991), the membership function of the fuzzy number \( Z \) can be obtained using the formulas:

\[ Z_1 = c_1 - d_3 \]  
\[ Z_3 = c_3 - d_1 \]

(13)  
(14)

That is, the fuzzy number \( Z \) is evaluated as:

\[ Z = [z_1, z_3] = C - D = [c_1, c_3] - [d_1, d_3] = [c_1 - d_3, c_3 - d_1] \]

(15)

If the fuzzy number \( Z \) is expressed using an \( \alpha \)-cut level set:

\[ Z(\alpha) = [z_1(\alpha), z_3(\alpha)] = [c_1(\alpha) - d_3(\alpha), c_3(\alpha) - d_1(\alpha)] \]

(16)

If \( C \) and \( D \) are triangular fuzzy numbers, the fuzzy numbers are:

\[ C = [c_1, c_2, c_3] \]  
\[ D = [d_1, d_2, d_3] \]

(17)  
(18)

So, the fuzzy number \( Z \) is:

\[ Z = [z_1, z_2, z_3] = [c_1 - d_3, c_2 - d_2, c_3 - d_1] \]

(19)

Slope failure occurs when \( C \) exceeds \( D \); that is, when \( Z \leq 0 \). In contrast, the event \( Z > 0 \) implies safety or reliability (Figure 3). Note that \( Z \) will also be a triangular fuzzy number in fuzzy arithmetic, as in Kaufmann and Gupta (1991).

To quantify reliability in fuzzy number analysis, the fuzzy reliability index (FR) is suggested (Shrestha and Duckstein, 1998), which is similar to the probabilistic reliability index but indicates the probability that the membership function of the safety margin is larger than 0. This is evaluated as the ratio of the total area of the membership of \( Z \) to the area of the membership for which \( Z \) is greater than 0 (Figure 3):

\[ FR = \frac{\int_{z_1}^{z_3} \mu_z(z) \, dz}{\int_{z_1}^{z_3} \mu_z(z) \, dz} \]

(20)

where \( \mu_z(z) \) is the membership function of fuzzy number \( Z \).

The FR can be evaluated on the basis of the membership function values of the capacity and the demand. Based on the relative location of the membership function (or membership function values) of the capacity and demand, three different cases are suggested to compute the fuzzy reliability index.

Case 1. If the membership function of the capacity is always greater than the membership function of the demand, and there is no overlap between membership functions (that is, \( z_3 \geq 0 \) or \( c_3 \geq d_3 \)), then the FR shows a maximum value, i.e., 1 (Figure 4). This is the absolute safe case, since the capacity is always greater than the demand.

Case 2. If the membership function of the demand is always greater than the membership function of the capacity, and there is no overlap between membership functions (that is, \( z_3 \leq 0 \) or \( c_3 \leq d_1 \)), then the FR shows a minimum value, i.e., 0 (Figure 5).
Case 3. If the membership functions of the capacity and the demand do overlap (that is, $z_1 \leq 0$ and $z_3 \geq 0$), then the FR is between 0 and 1, and is obtained from Eq. (20) (Figure 3).

5. Case study

In order to check the feasibility and validity of the methods proposed in the present study, these methods were applied to a practical example. A slope at a highway construction site in the Cheongwon area (southern part of Korea) was selected, and a detailed field investigation was carried out. The dip direction and dip angle of the slope were 325° and 65°, respectively, and its height was 40.8 m. The slope was composed of Cretaceous igneous rock, the density of which was determined to be 2.65 g/cm³ from a laboratory test. In the design stage, the slope was evaluated to be stable for planar failure in the deterministic analysis, but planar failure did in fact occur during construction.

Approximately 350 discontinuity data were obtained from a scanline survey. Four discontinuity sets were recognized and their representative orientations were 65/152 (J1), 30/320 (J2), 25/094 (J3) and 68/190 (J4), respectively. Stereonet analysis showed that one of these discontinuity sets (J2) was kinematically unstable for planar failure when the friction angle was smaller than 30° (Figure 6). Therefore, the discontinuity set (J2) was selected for further stability study.

In addition, a direct shear test was carried out in order to acquire the shear strength parameter for discontinuity. Based on the 11 direct shear test results (Figure 7), the friction angle ranged from 20.9° to 46.3°, and their mean and standard deviation values were 34.6 and 8.2, respectively. However, since the number of the direct shear tests was limited and the true value of the friction angle could not be obtained, the friction angle was considered as an uncertain parameter.

The dip angle of the sliding plane was also considered as an uncertain parameter, because the dip direction and the dip angle values measured in the field investigation were quite different. However, the dip direction of the discontinuities was not considered as an uncertain parameter since the dip direction was not involved in the kinetic analysis of the rock slope stability. Based on the field investigation results, the mean and standard deviation values for the dip angle in the stability analysis were 30.0 and 3.0, respectively.

Cohesion was not considered in this slope stability analysis, on the basis of Hoek’s suggestion (2007).

5.1. Results of fuzzy based Monte Carlo simulation

5.1.1. Analysis results using triangular membership function

As can be seen, the friction angle obtained from the direct shear test includes a large amount of uncertainty. This uncertainty is usually caused by a lack of test results, thus preventing a precise understanding of the random properties for uncertain parameters. Therefore, in the present study, the friction angle was considered as a fuzzy number. The triangular membership function was chosen for the friction angle, because the fuzzy number should be normal and convex, and the triangular and trapezoidal shapes of the membership functions are most often used for representing fuzzy numbers (Klir and Yuan, 1995). Other shapes can be used, but the arithmetic operations for fuzzy numbers can be readily implemented with triangular and trapezoidal shapes of the membership functions. The minimum and maximum values of the membership function were determined as 20.9 and 46.3, respectively, on the basis of test results. A mean value of 34.6 was used as the core value in the membership function.

In addition, the dip angle of the sliding plane was also considered as a fuzzy number in the analysis. The triangular membership function was chosen, and the fuzzy numbers ranged from 27.0 to 33.0. A mean value of 30.0 was used as the core value in the membership function for the dip angle.

In the fuzzy based Monte Carlo simulation procedure, random numbers between 0 and 1 were generated, and these values were used as $\alpha$-level values. Subsequently, the $\alpha$-cut intervals were acquired from the
membership functions of the friction angle and dip angles, and then the elements values between the lower and upper bounds of the cut intervals were randomly selected. These values were used as input values in the performance function to evaluate the factor of safety. In addition, repeated calculations were implemented with randomly generated dip angles, friction angles, and performance functions, as suggested by Hoek and Bray (1981) and Wyllie and Mah (2004). By repeating this process, a large number of factors of safety were obtained. The distribution of factors of safety obtained from the fuzzy based Monte Carlo simulation is given in Fig. 8. The probability of failure obtained from the fuzzy based Monte Carlo simulation was 0.206.

5.1.2. Analysis results using trapezoidal membership function

In order to determine the influence of the membership function shape in the input parameters, the trapezoidal membership function was used for both the friction angle and dip angle. For the friction angle, the fuzzy number ranged from 20.9 to 46.3, which were same values used in the triangular membership function. However, the core of the membership function was considered as the interval [31.2, 38.0]. In the case of the dip angle, the minimum and maximum values of the membership function were determined as 27.0 and 33.0. For the core values of the dip angle, the interval [29, 31] was selected. The calculation was then repeated with the trapezoidal membership function, and, subsequently, the distribution of the factors of safety was obtained (Figure 9). The probability of failure obtained from the trapezoidal function was 0.216, and this value was similar to the probability of failure obtained from the triangular function (0.206).

5.2. Results of fuzzy based reliability index analysis

5.2.1. Analysis results using triangular membership function

To further evaluate the probability of failure, a fuzzy based reliability index analysis was also applied. The friction angle and the dip angle of the sliding plane were considered as triangular fuzzy numbers, whose minimum, maximum, and core values were 20.9, 46.3, and 34.6, respectively, i.e. TFN (20.9, 34.6, 46.3). In the case of the dip angle, TFN (27.0, 30.0, 33.0) was defined as the membership function.

The capacity and demand for plane failure in the rock slope stability are evaluated as in (Hoek and Bray, 1981; Wyllie and Mah, 2004):

\[ C = W \cos \theta \tan \phi \]  (21)

\[ D = W \sin \theta \]  (22)

where W is the weight of the sliding block, \( \theta \) is the dip angle of the slide plane, and \( \phi \) is the friction angle of the sliding plane. Therefore, using triangular fuzzy numbers for the friction angle and the dip of the sliding plane, the capacity and demand are evaluated as:

\[ C = \text{TFN}[758.3, 1667.9, 3076.9] \]  (23)

\[ D = \text{TFN}[1289.7, 1395.9, 1498.2] \]  (24)

Then the fuzzy number of the safety margin is evaluated from the fuzzy arithmetic of the fuzzy number mentioned previously:

\[ Z = \text{TFN}[-739.9, 272.0, 1787.2] \]  (25)

The calculated FR is 0.802, meaning that the probability of failure is 0.198. This result is similar to the results of fuzzy based Monte Carlo simulation with the triangular membership function, which was obtained as 0.206.

5.2.2. Analysis results using trapezoidal membership function

Also, in order to check the effect of the membership function shape, the trapezoidal membership function was utilized to evaluate the probability of failure in the fuzzy based reliability analysis. The trapezoidal membership functions for the friction angle and dip, which were TrFN(20.9, 31.2, 38.0, 46.3) and TrFN(27.0, 29.0, 31.0, 33.0), respectively, which are same membership functions used in fuzzy based Monte Carlo simulation. Subsequently, using the trapezoidal fuzzy numbers for the friction angle and the dip of the sliding plane, the capacity and demand are obtained as:

\[ C = \text{TrFN}[758.3, 1562.9, 1769.5, 3076.9] \]  (26)

\[ D = \text{TrFN}[1289.7, 1360.9, 1430.5, 1498.2] \]  (27)

Consequently, the fuzzy number of the safety margin, which is obtained from the fuzzy arithmetic operation, is:

\[ Z = C - D = \text{TrFN}[-739.9, 132.4, 408.6, 1787.2] \]  (28)

The fuzzy reliability index is evaluated as 0.776, meaning that the probability of failure is 0.224 (Figure 11). The probabilities of failure evaluated from the fuzzy based reliability approach are similar to the probabilities of failure from fuzzy based Monte Carlo simulation. Therefore, in the case of the approaches using fuzzy set theory, fuzzy based Monte Carlo simulation and the fuzzy based reliability
index show consistent analysis results, since the probabilities of failure obtained from the fuzzy approach range from 0.198 to 0.224.

5.3. Results of the deterministic and the probabilistic analysis

5.3.1. Analysis results of the deterministic approach

In order to compare the results of the suggested approaches, a deterministic analysis based on the limit equilibrium approach was carried out. The deterministic analysis was performed with the same input values for all the deterministic parameters (density, slope height, and slope angle) and the mean values of the distributions for the random parameters (friction angle and dip angle of the sliding plane) used in the previous analysis (Table 1). The performance function suggested by Hoek and Bray (1981) and Wyllie and Mah (2004) was used to evaluate the factor of safety, and RocPlane software (Rocscience, 2003) was used for the deterministic analysis. The factor of safety was evaluated as 1.21, and it was analyzed as stable in the deterministic analysis (Figure 12). However, this result does not correspond to the results of the previous analysis because the deterministic analysis does not reflect the variability and uncertainty in the input parameters.

5.3.2. Analysis results of the probabilistic approach

A probabilistic analysis was also carried out using the procedure proposed by Park et al. (2005). In this procedure, the same performance function used in the previous analysis and the Monte Carlo simulation were employed in order to evaluate the probability of failure. The friction angle and dip angle of the sliding plane were considered as random variables in the probabilistic approach. A mean value of 34.6 and a standard deviation of 8.2 were used for the friction angle, but the probability density function could not be determined from the histogram plot (Figure 7) of the test results, due to the limited number of tests. It is desirable to obtain as many samples as possible and determine the probability density function from the data. However, in engineering geology, there are some practical and financial limitations to the amount of data which can be obtained, and consequently, it is often necessary to make estimates on the basis of judgment, experience, or results published by others (Hoek, 2007). Since previous research works used a normal distribution as the probability density function of the friction angle (Mostyn and Li, 1993; Nilsen, 2000; Pathak and Nilsen, 2004; Park et al., 2005), a normal distribution was chosen for the probability density function of the friction angle in this study. With the dip angle, the

<table>
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<th>Input parameters in the deterministic and probabilistic analysis.</th>
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<td>Probabilistic parameters</td>
</tr>
<tr>
<td>Input parameter</td>
<td>Value</td>
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<tr>
<td>Slope direction (dip/dip direction)</td>
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<td>Slope height (m)</td>
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<tr>
<td>Density of rock (g/cm³)</td>
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![Fig. 10. Membership function of (a) capacity and demand and (b) safety margin with triangular fuzzy numbers.](image)

![Fig. 11. Membership function of (a) capacity and demand and (b) safety margin with trapezoidal fuzzy numbers.](image)
mean value of 30.0 and a standard deviation of 3.0 were used, and a triangular distribution was utilized as the probability density function, due to the limitation of information. In order to obtain the probability of slope failure, a total of 16,000 repeated calculations were carried out in the Monte Carlo simulation procedure. Fig. 13 shows the distributions of the factors of safety that resulted from the probabilistic analysis. The probability of failure was evaluated as 0.289, which is quite high. The probability of failure obtained from the probabilistic analysis is somewhat higher than the probability of failure evaluated from the fuzzy approaches.

However, the selected probability density function for random variable may cause the miscalculation of the probability of failure due to the limitation of data. Therefore, the probability of failure is evaluated by a nonparametric analysis method, which does not rely on data drawn from a given probability distribution. The point estimate method is used in this analysis. The point estimate method, originally proposed by Rosenblueth (1975, 1981) and Harr (1987), is a simple but powerful technique for evaluating the reliability of a system. Like FOSM, the point estimate method does not require knowledge of the probability density function for input, but it can still be accurate in many practical situations despite its simplicity (Christian and Baecher, 2002). The probability of slope failure calculated by the point estimate method was 0.274. The probability of failure evaluated from the point estimate method is similar to the probability obtained from the probabilistic Monte Carlo simulation, but slightly higher than the fuzzy approaches.

5.3.3. Effects of uncertainty in the analysis

The coefficient of variation (COV) for the friction angle used in this study was calculated as 23.3%, which is quite high compared to previous research results, which showed a COV of 10% for the friction angle (Schultze, 1975; Cherubini, 2000; Babu and Mukesh, 2003). This means that the dispersion of the direct shear test results used in the present study was too large, i.e. the randomly generated friction angle from the Monte Carlo simulation ranged from 10.0 to 59.2 in the confidence interval of 99.8%. Consequently, the uncertainty of the friction angle was too large, and in the Monte Carlo simulation, too small a value or too high a value for the friction angle could be generated and used in the factor of safety calculation. Thus, the probability of failure obtained from the probabilistic analysis can be overestimated when the probability of failure is evaluated on the basis of the random properties of input values obtained from a limited amount of data.

If a sufficient amount of data can be obtained for random variables or additional data can be provided, the uncertainties could be controlled and the variability of data could be reduced. Therefore, in order to determine the influence of uncertainty on the input parameters and to control the uncertainty in the evaluation of the probability of failure, a truncated normal distribution, with the COV reduced to 10% and the minimum value limited to 20.9, was used for the friction angle in the probabilistic analysis. The probability of failure was then recalculated with the truncated normal distribution. Fig. 14 shows the results of the analysis, wherein the evaluated
probability of failure was reduced to 0.242. In addition, the probability of failure evaluated from the point estimate method with the variability-reduced data is 0.178. The results of the probabilistic analysis with variability-reduced data are more similar to the results of the fuzzy approaches. Consequently, the fuzzy uncertainty caused by incomplete information is not handled satisfactorily by the probabilistic approach. However, the analysis results obtained from fuzzy based Monte Carlo simulation and fuzzy based reliability are in reasonable agreement with the results of the probabilistic analysis when the variability-reduced data was used.

6. Conclusions

The present study has proposed a useful approach for properly addressing the fuzzy uncertainties caused by incomplete information and for using fuzzy set theory to evaluate the reliability of a slope. In this study, uncertain parameters are taken into account as fuzzy numbers, and uncertainties in the input variables are handled in fuzzy based Monte Carlo simulation and fuzzy based reliability approach in order to obtain the probability of slope failure.

In order to verify the feasibility and validity of the proposed approaches, they were applied to a practical example. The probabilities of failure obtained from fuzzy based Monte Carlo simulation and fuzzy based reliability index were 0.206 and 0.198 (respectively) when the triangular membership function was used, and 0.216 and 0.224 (respectively) when the trapezoidal membership function was used. Deterministic and probabilistic analyses were also carried out for comparison with the results of the proposed approaches. The results of the deterministic analysis showed that the slope was stable for plane failure, since the factor of safety was evaluated as 1.21. This is because the deterministic analysis uses only mean or representative values for input parameters and fails to consider uncertainties in the input parameters. As mentioned previously, the slope experienced planar failure during construction, so the deterministic analysis failed to indicate a slope failure condition. The probabilistic analysis results using the data incorporating fuzzy uncertainties showed 0.289 in Monte Carlo simulation and 0.274 in point estimate method, both of which were somewhat higher than the results obtained from the fuzzy approaches. On the other hand, when data with reduced variability in terms of friction angle was used, the probability of plane failure was evaluated as 0.242 and 0.178, respectively. The analysis results from the variability-reduced data are similar to the analysis results from the fuzzy approaches. Therefore, fuzzy based Monte Carlo simulation and the fuzzy based reliability index provide consistent measures of safety for slopes. In addition, fuzzy set theory seems to have effectively addressed fuzzy uncertainties.

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