Distributed Federated Kalman Filter Fusion Over Multi-Sensor Unreliable Networked Systems

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Abstract—This paper is concerned with the problem of distributed federated Kalman filter fusion (DFKFF) for a class of multi-sensor unreliable networked systems (MUNSs) with uncorrelated noises. An optimal DFKFF algorithm of MUNSs without buffer is presented, and rigorously proved to be equivalent to centralized optimal Kalman filter fusion (COKFF) algorithm of MUNSs without buffer. Finite length buffers deal with measurement delay or loss, and a suboptimal DFKFF algorithm of MUNSs with finite length buffers is proposed based on the optimal local Kalman filter with a buffer of finite length for each subsystem. Compared with COKFF algorithm of MUNSs with buffers, the proposed DFKFF algorithm of MUNSs with buffers has stronger fault-tolerance ability. Two simulation examples are given to illustrate the effectiveness of the proposed approaches.

Index Terms—Buffers, Distributed federated Kalman filter fusion (DFKFF), measurement delay or loss, multi-sensor unreliable networked systems (MUNSs).

I. INTRODUCTION

MULTI-SENSOR information fusion and federated filter technologies have been researched and generally applied to civil and military fields, such as localization, guidance, navigation, target tracking, fault diagnosis, spacecraft attitude estimation, etc. There are many information fusion methods for a class of networked multi-sensor fusion systems (NMFSs) [1], [2]. Compared with the traditional fusion systems, the distributed NMFSs possess many advantages, for example, simpler installation, lower cost, easier maintenance and more flexible architectures. Due to inherent properties of NMFSs, there are still some challenges of data transmission delay, fading and loss, the mathematical complexity created by autocorrelated and cross-correlated noises, constrained bandwidth. To overcome these challenges, the distributed fusion problems for a class of multi-sensor unreliable networked systems (MUNSs) have attracted more and more researchers’ attention.

With the development of the technology of aerospace, Carlson [3], [4] invented the federated filter, which had been widely used in aerospace fields. A modified federated Kalman filter in [5] and an optimized information sharing federated Kalman filter algorithm in [6] were designed, which greatly improved the navigation performance of combined system. Based on two principles of state vector augmentation and rejection of part of the information, a synthesis method federated filter was developed in [7].

In order to meet the requirements of higher accuracy, the distributed fusion problems for multi-sensor systems have received extensive application. So far, various forms of distributed fusion have been studied in literatures, such as optimal distributed fusion with the linear transformed data [8], optimal sequential distributed fusion [9], optimal distributed Kalman filtering fusion [10], distributed weighted fusion [11]–[13], [20], distributed covariance intersection fusion [14], distributed multi-rate fusion [15], [16], distributed Kalman filter [17]–[19], etc. Two optimal distributed fusion algorithms were presented in [8] based on linear transformation of the original measurement at each sensor. The problems of optimal distributed fusion in a sequential form for multi-sensor linear systems with cross-correlated noises were researched in [9]. A modified Kalman filtering optimal distributed fusion with feedback for linear dynamic systems with cross-correlated noises was proposed in [10], and it was proved that it was equivalent to the centralized Kalman filtering fusion by all sensor measurements under a mild condition. Using the maximum likelihood fusion criterion under the assumption of standard normal distribution, an optimal multi-sensor weighted information two-layer fusion Kalman filter was designed in [11]. For one-step autocorrelated and two-step cross-correlated process noise and measurement noises, the fusion estimation of distributed weighted robust Kalman filter was proposed in [12]. In the condition of the worst-case preserved systems with noise variances of conservative upper bounds, distributed robust weighted fusion Kalman filters for multi-sensor time-varying systems with uncorrelated noise were designed by using the principle of minimax robust estimation and the optimal estimation rule of unbiased linear minimum variance [13]. Distributed covariance intersection fusion algorithm for multi-sensor stochastic uncertain systems with correlated noises was proposed in [14], and compared with the algorithm in [12]. The authors concerned state estimation and data fusion problems of wireless multi-rate sensor networked systems in [15]. The multi-sensor fusion estimation problem for wireless sensor networks with nonuniform estimation rates was investigated in [16]. A distributed implementation of the Kalman filter [17] and a distributed partition-based...
estimation scheme based on Kalman filter [18] were proposed for distributed monitoring and control of large-scale systems with sensor networks in the case where the sensors can observe only a small piece of the overall state. A direct connection of the covariance debiasing methodology for the distributed Kalman filter to the federated Kalman filter was shown in [19]. It explained that how the distributed Kalman filter is related to the federated Kalman filter. However, they are not involved in delay and packet loss in the mentioned literatures of this paragraph.

To deal with the delays and available data packets, various forms of processing methods have been used in literatures. A novel stochastic model was proposed to describe the transmission delays and packet dropouts in [20] and [21]. How to design a filter that ensures performance despite the presence of measurement delays is an important problem to be considered. Therefore, the problems of estimating a class of stochastic systems observed by a single delay-tolerant sensor were investigated in [22] and [23] by using the innovation analysis approach and linear matrix equality approach. Multiple binary random variables with known probabilities were introduced to model random delays in the measurements, and a robust distributed state fusion Kalman filter was designed for a class of MUNSs in [24]. One redundant channel in the communication network was introduced in [25] for the first time to reduce the effect of both quantization and randomly occurring packet dropouts in the two channels, and it was compared with the scenario of only single channel being used. In [26], the researchers introduced a discrete-time Markov chain to capture the signal losses, and given the design and convergence of distributed estimation algorithms under various uncertainties. Modeling the arrival of the observation as a random process, the problem of Kalman filtering with intermittent observations was performed in [27]. Processing quantization errors and successive packet dropouts to a unified framework, a new distributed finite-horizon filter technique [28] was proposed to satisfy the prescribed average filter performance constraint. Finite length buffers was presented to deal with measurements delay or loss [29], [30]. The authors studied the state estimate problem for single channel with a buffer of finite length, and the relationship between the stability and the packet arrival process was analyzed in [29]. The researchers extended the state estimation algorithm from single channel to multi-channel, and centralized fusion estimation problem for MUNSs was presented in [30]. A similar buffering technique has been presented in [31], which is applicable to the case with packet delays and losses. Analyzing the stability of networked filter and coping with delays by the proposed federated filter, a method of data fusion for a networked system with packet losses and delays was considered in [32]. Delay and loss of measurement in the transmission from sensor to filter are both modeled by a Bernoulli distributed random sequence, and the problem of robust filtering was researched for a class of MUNSs in [33].

When data packets were lost on transmission, it was solved by conventional Kalman predication technology, and the proposed algorithm in [32, Sec. III-C] was applied to deal with the delayed data packets. In this paper, we use buffers and a random process of modeling the arrival of the observation in [27] to deal with measurement delay and loss, respectively. Under a mild condition, a distributed fusion Kalman filter was proved to be equivalent to the centralized fusion Kalman filter [10], but the proof of equivalence in this paper is a step-by-step and rigorous process by using the mathematical induction method without restriction. Centralized optimal Kalman filter fusion (COKFF) algorithms of MUNSSs with or without buffer in [30] are used to compare with our proposed algorithms.

Inspired by the idea of framing in Fig. 1, all measurements are sent to local estimators through communication channels from sensors. The system suffers measurements delay or loss because of the unreliability of MUNSSs. Each sensor makes fault detection. The classical methods for fault detection employed WSSR [34] and U verification [35]. If any sensor subsystem is faulty by detection, it is isolated and restored [11]. Otherwise, their estimates and variances are delivered to the main filter. An optimal distributed federated Kalman filter fusion (DFKFF) algorithm of MUNSSs without buffer is proposed when buffers are not considered. Finite length buffers deal with measurements delay or loss [29], [30], and DFKFF algorithm of MUNSSs with buffers is presented.

Motivated by the aforementioned analysis, the purpose of this paper is to propose DFKFF algorithms for a class of MUNSSs with uncorrelated noises. Firstly, we present an optimal DFKFF algorithm of MUNSSs without buffer, which rigorously proved to be equivalent to COKFF algorithm of MUNSSs without buffer. Secondly, MUNSSs suffer measurement delay or loss because of their unreliability. Based on the optimal local Kalman filter with a finite length buffer for each subsystem, DFKFF algorithm of MUNSSs with finite length buffers has been given. Finally, comparison studies are carried out in simulation results to illustrate the effectiveness of the proposed algorithms.

The main contributions of this paper are highlighted as follows:

1) The proposed DFKFF algorithm of MUNSSs without buffer is optimal and recursive, and is equivalent to COKFF algorithm of MUNSSs without buffer. Rigorous proof procedure and simulation verification are given.

2) It takes into account unreliability of MUNSSs by finite length buffers and a random process to deal with measurement delay and loss. Based on the optimal local Kalman filter with a buffer of finite length, the proposed DFKFF algorithm of MUNSSs with buffers is suboptimal and recursive, and has stronger fault-tolerance ability.
This paper is organized as follows. Section II states the multisensor linear discrete-time stochastic control system model and some problem statements. The optimal DFKFF algorithm of MUNSs without buffer is described in Section III. Section IV is dedicated to the DFKFF algorithm of MUNSs with buffers. Corresponding simulation results are provided in Section V. And this study is concluded in Section VI.

Notations: The superscript “T” denotes the transpose, and \( I_n \) represents the identity matrix of \( n \) dimensions. \( E \{ \cdot \} \) is the mathematical expectation. \( \text{diag}\{\cdot\} \) denotes the diagonal matrix. The superscripts \( i, j, f \) are the matrix or variable related to the \( i \)th local estimator, DFKFF algorithm of MUNSs without buffer and DFKFF algorithm of MUNSs with buffers, respectively. \( p(\cdot) \) is a probability distribution.

II. MODEL AND PROBLEM STATEMENTS

Consider a discrete-time linear stochastic system with \( N \) sensors described by the following state-space model:

\[
\begin{align*}
x_{k+1} &= A_k x_k + w_k \\
z^i_k &= C_{i,k} x_k + D_{i,k} v_{i,k}, \quad i = 1, 2, \ldots, N
\end{align*}
\]

(1)

where \( x_k \in \mathbb{R}^n \) is the state vector, \( z^i_k \in \mathbb{R}^{m_i} \) (\( i = 1, 2, \ldots, N \)) are measurement vectors, \( A_k \in \mathbb{R}^{n \times n}, C_{i,k} \in \mathbb{R}^{m_i \times n}, \) and \( \begin{align*}D_{i,k} \in \mathbb{R}^{m_i \times m_i,} \end{align*} \) are known matrices. \( w_k \in \mathbb{R}^r \) and \( v_{i,k} \in \mathbb{R}^{m_i} \) (\( i = 1, 2, \ldots, N \)) are uncorrelated zero-mean Gaussian white noises, which satisfy

\[
E \{ [w_k \ v_{i,k}]^T [w_l \ v_{i,l}] \} = \delta_{kl} \text{diag} \{ Q_k, R_{i,k} \}
\]

(2)

where \( Q_k \geq 0 \) and \( R_{i,k} > 0 \) are process and measurement noise covariances, respectively, and \( \delta_{kl} \) is the Kronecker function

\[
\delta_{kl} = \begin{cases} 1 & \text{if } k = l \\ 0 & \text{if } k \neq l. \end{cases}
\]

Assumption 1: The initial state \( x_0 \) is independent of \( w_k \) and \( v_{i,k} \) (\( i = 1, 2, \ldots, N \)), and

\[
E \{ x_0 \} = \hat{x}_0, \quad E \{ [x_0 - \hat{x}_0] [x_0 - \hat{x}_0]^T \} = P_0.
\]

Assumption 2: \((A_k, C_{i,k})\) is observable, and \((A_k, Q_k^{1/2})\) is controllable.

All measurements are sent to local estimators through communication channels. Time stamping is requisite to receive measurements at local estimators under measurements delay or even be lost because of unreliability of MUNSs. Suppose that measurements are sent to local estimators from sensors with negligible quantization effects. Buffers of finite length are proposed to deal with measurement delay, and a term of measurement noises [27] processes measurement loss. And each sensor makes fault detection. If any sensor subsystem is faulty by detection, it is isolated and restored. Otherwise, their estimates and variances are delivered to the main filter. Then, a distributed method of calculating the optimal fusion estimate and the error covariance based on DFKFF of MUNSs architecture is presented in Fig. 1.

Each \( z^i_k \) is delayed by \( d^i_k \) times, where \( d^i_k \) is a random variable depicted by a probability mass function \( f_i \) [29], [30]

\[
f_i(j) = \Pr \{ d^i_k = j \}, \quad j = 0, 1, 2, \ldots, i = 1, 2, \ldots, N.
\]

(3)

To keep it simple, it is assumed that \( d^i_{k1} \) and \( d^i_{k2} \) are independent when \( i_1 \neq i_2 \) or \( k_1 \neq k_2 \). It is further assumed that \( d^i_k \) is independent of \( w_k, v_{i,k} \) and the initial state \( x_0 \).

There are two goals of this research described as follows:

1) Firstly, an optimal DFKFF algorithm of MUNSs without buffer will be proposed, which is equivalent to COKFF algorithm of MUNSs without buffer;

2) Secondly, a suboptimal DFKFF algorithm of MUNSs with buffers will be presented, which has stronger fault-tolerance ability.

III. DFKFF AND COKFF ALGORITHMS

OF MUNSs WITHOUT BUFFER

At first, let \( \gamma^i_{k,k} \) be an indicator function for \( z^i_k \) at time \( k \) as follows:

\[
\gamma^i_{k,k} = \begin{cases} 1 & \text{if } z^i_k \text{ arrives at time } k \\ 0 & \text{otherwise}. \end{cases}
\]

(4)

More formally, \( z^i_{k,k} \) is the observed measurement value at time \( k \), and \( z^i_{k,k} \) can be written as

\[
z^i_{k,k} = \gamma^i_{k,k} z^i_k = \gamma^i_{k,k} C_{i,k} x_k + \gamma^i_{k,k} D_{i,k} v_{i,k}.
\]

(5)

Then, the measurement noise \( v^i_k \) is defined as a random process in the following term [27]:

\[
p \left( v_{i,k} | \gamma^i_{k,k} \right) \sim \begin{cases} N(0, R_{i,k}) & \text{if } \gamma^i_{k,k} = 1 \\ N(0, \rho^2 I) & \rho \to \infty, \quad \gamma^i_{k,k} = 0. \end{cases}
\]

(6)

A. DFKFF Algorithm of MUNSs Without Buffer

According to the federated Kalman filter fusion reset algorithm [6], formulas (4) and (5), DFKFF algorithm without buffer can be implemented as follows:

Step 1) Information sharing process. \( \hat{x}^i_{k-1|k-1} \) and \( P^i_{k-1|k-1} \) are the state estimate and the error covariance of the \( i \)th local estimator at time \( k - 1 \), respectively. In accordance with the following principle of information sharing coefficient, the process information \( Q_k^{-1} \) and \( P^i_{k-1|k-1}^{-1} \) have been allocated between each subestimator and the main filter

\[
\begin{align*}
Q_k^{-1} &= Q_{k-1}^{-1} \beta^{-1}_i \\
\beta^i_{k-1|k-1} &= P^i_{k-1|k-1}^{-1} \beta^{-1}_i \\
\hat{x}^i_{k-1|k-1} &= \hat{x}^i_{k-1|k-1}, \quad i = 0, 1, \ldots, N
\end{align*}
\]

(7)

where \( \beta_i \geq 0 \) is an information sharing coefficient satisfying the condition of conservation of information

\[
\sum_{i=1}^{N} \beta_i = 1.
\]

(8)
Step 2) Time update. \( \hat{x}_{k|k-1} \) and \( P_{k|k-1} \) are the one-step prediction state estimate and the one-step prediction error covariance of the \( i \)th local estimator, respectively:

\[
\begin{align*}
\hat{x}_{k|k-1} &= A_{k-1} \hat{x}_{k-1|k-1} \\
P_{k|k-1} &= A_{k-1} P_{k-1|k-1} A_{k-1}^T + Q_{k-1} \quad i = 0, 1, \ldots, N. \tag{9}
\end{align*}
\]

Step 3) Measurements update. \( K_k \) is the Kalman gain of the \( i \)th local estimator:

\[
\begin{align*}
K_k &= P_{k|k-1} C_{i,k} (C_{i,k} P_{k|k-1} C_{i,k}^T + R_{i,k} D_{i,k}^T)^{-1} \\
\tau_{i,k} &= \hat{x}_{k|k-1} + K_k (z_{k|k} - C_{i,k} \hat{x}_{k|k-1}) \\
P_{k|k} &= P_{k|k-1} - K_k C_{i,k} P_{k|k-1}, \quad i = 0, 1, \ldots, N. \tag{10}
\end{align*}
\]

Step 4) Information fusion:

\[
\begin{align*}
P_i^f_{k|k} &= \left( \sum_{i=1}^{N} (P_{k|k}^{-1}) \right)^{-1} \\
\hat{x}_{i,k} &= P_i^f_{k|k} \sum_{i=1}^{N} (P_{k|k}^{-1}) \hat{x}_{k|k}. \tag{11}
\end{align*}
\]

Remark 1: In the above algorithm, \( P_i^f_{k|k} \) and \( \hat{x}_{i,k} \) are reset at each time \( k \) in Step 1, and the information sharing coefficient of the main filter \( \beta_m \) is set to zero [5]. In general, each sub-estimator receive the feedback information \( \tau_{i,k} \) from the main filter, and the main filter does not receive the feedback information. That is to say, the information of the main filter is not reset.

B. COKFF Algorithm of MUNSs Without Buffer

The augmented measurement vectors of COKFF algorithm of MUNSs without buffer [30] can be expressed as follows:

\[
\begin{align*}
z_k &= \begin{bmatrix} z_{1,k}^T, z_{2,k}^T, \ldots, z_{N,k}^T \end{bmatrix}^T \\
C_k &= \begin{bmatrix} C_{1,k}^T, C_{2,k}^T, \ldots, C_{N,k}^T \end{bmatrix}^T \\
D_k &= \text{diag}\{D_{1,k}, D_{2,k}, \ldots, D_{N,k}\} \\
v_k &= \begin{bmatrix} v_{1,k}^T, v_{2,k}^T, \ldots, v_{N,k}^T \end{bmatrix}^T \\
R_k &= \text{diag}\{R_{1,k}, R_{2,k}, \ldots, R_{N,k}\}. \tag{12}
\end{align*}
\]

Then, COKFF algorithm of MUNSs without buffer can be given by

\[
\begin{align*}
\hat{x}_{k|k-1} &= A_{k-1} \hat{x}_{k-1|k-1} \\
P_{k|k-1} &= A_{k-1} P_{k-1|k-1} A_{k-1}^T + Q_{k-1} \\
\tau_{i,k} &= \hat{x}_{k|k-1} + K_k (z_k - C_{i,k} \hat{x}_{k|k-1}) \\
P_i^f_{k|k} &= P_i^f_{k|k-1} - K_k C_{i,k} P_{k|k-1}, \quad i = 0, 1, \ldots, N. \tag{13}
\end{align*}
\]

where \( \hat{x}_{k|k-1} \) and \( P_{k|k-1} \) are the one-step prediction state estimate and the one-step prediction error covariance of COKFF algorithm of MUNSs without buffer, respectively. \( K_k, \hat{x}_{k|k} \) and \( P_{k|k} \) are the Kalman gain, the state estimate and the error covariance of COKFF algorithm of MUNSs without buffer at time \( k \), respectively.

C. Equivalence of Two Algorithms

To show that DFKFF algorithm of MUNSs without buffer is equivalent to COKFF algorithm of MUNSs without buffer, we need to prove that (14), (15), and (24), (25) are equivalent, respectively. Meanwhile, we derive the following theorem for the equivalence of the two algorithms:

Theorem 1: DFKFF algorithm and COKFF algorithm of MUNSs without buffer are equivalent, thus DFKFF algorithm of MUNSs without buffer is optimal.

Proof: See A.1 in Appendix. \( \blacksquare \)

Remark 2: Using the principles of variance upper bound in [4], the initial error covariance matrix of each sub-estimator is enlarged to ignore the error cross-covariance matrix between various sub-estimators. Their correlation is eliminated. In DFKFF algorithm of MUNSs without buffer, after the completion of Step 4, various sub-estimators will be reset by the main filter. Then the error covariance matrix of each sub-estimator become relevant once more. Again using the principles of variance upper bound, this correlation will be eliminated.

IV. DFKFF Algorithm of MUNSs With Buffers

At first, it is assumed that local estimators employ a buffer with length \( L \geq 2 \) at each time \( k \), and are able to retrieve all available measurement data packets up to time \( k - L + 1 \). Measurements are not delayed if \( L = 1 \) and delayed if \( L \geq 2 \) and \( k \geq L \). Let \( \gamma_{i,k} \) be an indicator function for \( z_{i,k} \) at time \( k \) \((t = k - L + 1, \ldots, k)\) as follows:

\[
\gamma_{i,k} = \begin{cases} 
1 & \text{if } z_{i,k} \text{ arrives before or at time } k \\
0 & \text{otherwise}
\end{cases}
\]

Stated more formally, \( z_{i,k} \) \((i = 1, 2, \ldots, N)\) are stored in the \((t + L - k)\)th position of the buffer at time \( k \), and \( \gamma_{i,k} \) can be written as

\[
z_{i,k} = \gamma_{i,k} z_{i,t} = \gamma_{i,k} C_{i,t} x_t + \gamma_{i,k} D_{i,t} v_{i,t}.
\]

Then, the measurement noise \( v_{i,t} \) is defined as a random process of modeling the arrival of the observation \([27]\)

\[
p(v_{i,t} | \gamma_{i,k}) = \begin{cases} 
N(0, R_{i,t}), & \gamma_{i,k} = 1 \\
N(0, \rho^2 I), & \rho \to \infty, \quad \gamma_{i,k} = 0
\end{cases}
\]

Remark 3: In this paper, if \( z_{i,k} \) is not received by local estimator \( i \) before or at \( k \), then it will be considered to be lost. If \( \gamma_{i,k} = 1 \), then \( \gamma_{i,k+t} = 1 \forall t \in \mathbb{N} \), which indicates that if \( z_{i,k} \) is received by local estimator \( i \) before or at time \( k \), it will be present all the time in the future. According to [30], an example of \( \gamma_{i,k} \) is vividly illustrated in Fig. 2.
At this point, without considering the reset, we design an optimal local Kalman filter estimator $i$ with a buffer of finite length in the following form:

$$
\hat{x}_{k|k}^i = \hat{x}_{k-1|k-1}^i + P_{k|k}^i (z_{k}^i - \hat{x}_{k|k}^i)
$$

where $Z_{k}^i = [z_{1|k}^i, z_{2|k}^i, \ldots, z_{n|k}^i]$ and $\gamma_k^i = [\gamma_{1|k}^i, \gamma_{2|k}^i, \ldots, \gamma_{I|k}^i]$. Without loss of generality, the error, the error covariance, one-step prediction error, one-step prediction error covariance, and some variables were defined as follows:

$$
\tilde{x}_{k|k}^i \triangleq x_k - \hat{x}_{k|k}^i
$$

$$
P_{k|k}^i \triangleq E \left( z_k^i - \tilde{x}_{k|k}^i \right) (z_k^i - \tilde{x}_{k|k}^i)^T
$$

$$
\tilde{x}_{k|k-1}^i \triangleq x_k - \tilde{x}_{k|k-1}^i
$$

$$
P_{k|k-1}^i \triangleq E \left( z_k^i - \tilde{x}_{k|k-1}^i \right) (z_k^i - \tilde{x}_{k|k-1}^i)^T
$$

$$
\tilde{x}_{l|l,k}^i \triangleq x_l - \tilde{x}_{l|l,k}^i
$$

$$
P_{l|l,k}^i \triangleq E \left( z_l^i - \tilde{x}_{l|l,k}^i \right) (z_l^i - \tilde{x}_{l|l,k}^i)^T
$$

Obviously, we have $\tilde{x}_{k|k}^i = \tilde{x}_{k|k}^i$ and $P_{k|k}^i = P_{k|k}^i$.

In the following section, an useful theorem is given to get the DFKFF algorithm with buffers for a class of MUNSSs. Theorem 2 and Definition 1 provide Steps 2 and 3 for the DFKFF algorithm of MUNSSs with buffers, respectively.

**Theorem 2:** Consider the $i$th subsystem $(i = 1, 2, \ldots, N)$ of system (1), based on the standard Kalman filter and (27)–(35), the $i$th optimal local Kalman filter estimate $\hat{x}_{k|k}^i$ and the error covariance $P_{k|k}^i$ are determined by

$$
\hat{x}_{k-1|k-1}^i = \hat{x}_{k-1|k-1}^i - A_{i,k} \tilde{x}_{k-1|k-1}^i
$$

$$
P_{k-1|k-1}^i = P_{k-1|k-1}^i - A_{i,k} \tilde{x}_{k-1|k-1}^i (A_{i,k})^T + C_{i,k} D_{i,k} R_{i,k} (C_{i,k})^T
$$

$$
\tilde{x}_{l|l,k}^i = x_l - \tilde{x}_{l|l,k}^i
$$

$$
P_{l|l,k}^i = P_{l|l,k}^i - \tilde{x}_{l|l,k}^i (A_{i,k})^T + C_{i,k} D_{i,k} R_{i,k} (C_{i,k})^T
$$

where $\tilde{x}_{k|0,k}^i = x_0$, $P_{k|0,k}^i = P_0$, and $t = \kappa_i, \ldots, k$. The $i$th optimal local Kalman filter estimate $\hat{x}_{k|k}^i$ and the error covariance $P_{k|k}^i$ are got by iterating $k - \kappa_i + 1$ times from $\tilde{x}_{k_i-1|k_i-1,k_i}$ and $P_{k_i-1|k_i-1,k_i}$, respectively.
Proof: See A.2 in Appendix.

Remark 6: \( \hat{x}_{i,k|k} \) and \( P_{i,k|k} \) are got by iterating \( k - n_i + 1 \) times from \( \hat{x}_{i,-1|k-1} \) and \( P_{i,-1|k-1} \) during the \( i \)th buffer at time \( k \), respectively. Then we have \( \hat{x}_{i,k} = \hat{x}_{i,k|k} \) and \( P_{i,k|k} = P_{i,k,k|k} \).

Based on the optimal local Kalman filter estimate \( \hat{x}_{i,k} \) from (39) and error covariance \( P_{i,k|k} \) from (42), we further research the DFKFF algorithm with buffers for a class of MUNSs with measurement delay or loss and uncorrelated noises.

**Definition 1:** For system (1), based on Theorem 2 and the federated filter criteria [5], the distributed federated Kalman filter fusion estimate \( \hat{x}_{i,k} \) and its corresponding error variance \( P_{i,k|k} \) are obtained by

\[
\hat{x}_{i,k|k} = \left[ \sum_{i=1}^{N} \left( P_{i,k|k}^{-1} \right) \right]^{-1} \left[ \sum_{i=1}^{N} \left( P_{i,k|k}^{-1} \right) \hat{x}_{i,k|k} \right]
\]

\[
P_{i,k|k} = \left[ \sum_{i=1}^{N} \left( P_{i,k|k}^{-1} \right) \right]^{-1}
\]

where \( \hat{x}_{i,k} = \hat{x}_{i,k|k} \) and \( P_{i,k|k} = P_{i,k,k|k} \).

According to DFKFF algorithm of MUNSs without buffer, Theorem 2 and Definition 1, DFKFF algorithm of MUNSs with buffers is given as follows:

DFKFF algorithm of MUNSs with buffers

1. **Step 1:** Information sharing process of initialization.
   
   \[
   \begin{align*}
   Q^i_0 &= Q_0 \beta_i^{-1} \\
   P_{0|0} &= P_{0|0} \beta_i^{-1} = P_0 \beta_2^{-1} \\
   \hat{x}_{0|0} &= \hat{x}_{0|0} = x_0, \quad i = 0, 1, \ldots, N
   \end{align*}
   \]

   where \( \beta_i \geq 0 \) follows (8).

2. **Step 2:** Buffer process of the \( i \)th optimal local Kalman filter (36)–(42), where \( \hat{x}_{i,k|k} = \hat{x}_{i,k} \) and \( P_{i,k|k|k} = P_{i,k|k,k|k} \), \( i = 0, 1, \ldots, N \).

3. **Step 3:** Information fusion.
   
   \[
   \begin{align*}
   P_{i,k|k} &= \left[ \sum_{i=1}^{N} \left( P_{i,k|k}^{-1} \right) \right]^{-1} \\
   \hat{x}_{i,k|k} &= P_{i,k|k} \sum_{i=1}^{N} \left( P_{i,k|k}^{-1} \hat{x}_{i,k|k} \right)
   \end{align*}
   \]

4. **Step 4:** Information sharing process of reset.
   
   \[
   \begin{align*}
   Q^i_k &= Q_k \beta_i^{-1} \\
   P_{i,k|k} &= P_{i,k|k} \beta_i^{-1} \\
   \hat{x}_{i,k|k} &= \hat{x}_{i,k|k} = \hat{x}_{i,k}, \quad i = 0, 1, \ldots, N
   \end{align*}
   \]

5. **Step 5:** Set \( k = k + 1 \) and go to Step 2.

Remark 7: Because various sub-estimators are not reset during buffers, namely, each sub-estimator do not receive the feedback information \( \hat{x}_{i,k}, P_{i,k|k}, Q_k \beta_i^{-1} \) from the main filter during buffers, it can derive the following formula:

\[
P_{i,k-1|k} \neq \sum_{i=1}^{N} \left( P_{i,k-1|k}^{-1} \right)^{-1}
\]

where \( P_{i,k-1|k} \) is expressed in [29]. In comparison to COKFF algorithm of MUNSs without buffer, the proposed DFKFF algorithm of MUNSs without buffer is suboptimal.

**Remark 8:** About the computational order of magnitude of DFKFF and COKFF algorithms, it is easily known that the DFKFF algorithm has the computational order of magnitude \( O((\sum_{i=1}^{N} m_i)^3) \) and COKFF algorithm has the computational order of magnitude \( O((\sum_{i=1}^{N} m_i)^3) \). The computational cost of DFKFF algorithm is lower than that of COKFF algorithm though the estimation accuracy has the loss in a small range.

**V. NUMERICAL EXAMPLES**

In this section, we use two examples to demonstrate the performance and effectiveness of our proposed algorithms.

**A. Example 1**

In this example, we consider the following target tracking MUNS with two local sensors, where the system parameters in (1) are described by:

\[
A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad C_1 = [1, 1], \quad C_2 = [1, 1.5] \\
D_1 = D_2 = [1], \quad Q = I_2, \quad R_1 = 1, \quad R_2 = 1.5
\]

where \( T = 0.01 \) is the sample period, two components of \( x_k \) is the position and velocity of the target.

For the simulation purpose, initial conditions are set as \( x_0 = [200, 10]^T \) and \( P_0 = \text{diag}(0.5, 0.1) \). Similar to [29], it is assumed that \( d_1 = d_2 = 3 \) and \( L_1 = L_2 = 5 \) in this paper, where \( d_i = E[d_i^2] \) denotes the mean value of the packet delay in the \( i \)th channel. In order to explain that both DFKFF and COKFF algorithms without buffer have no fault-tolerance, we first consider all sensors without faults condition. Then, it is assumed that the second sensor is faulty and the measurement equation is given by \( z^2_k = C_2 x_k + D_2 v_{2,k} + f(k) \) where \( f(k) \) satisfies \( f(k) = 0.2k(140 \leq k \leq 160) \) and \( f(k) = 0(0 < k < 140 \text{ or } k > 160) \). Traces of error covariances of local estimators (LEs) DFKFF and COKFF algorithms without buffer are shown in Fig. 4. When all sensors are faultless, root mean square errors (RMSEs) of positions and velocities of DFKFF and COKFF algorithms without buffer are depicted in Figs. 5 and 6, respectively. When the second sensor is faulty, Figs. 7 and 8 show RMSEs of positions and velocities DFKFF and COKFF algorithms without buffer.

From Fig. 4, we can see that traces of error covariances of DFKFF and COKFF algorithms without buffer are exactly the same, which are smaller than that of each LE without buffer. Meanwhile, it can be seen from Figs. 5 and 6 that RMSEs of positions and velocities DFKFF and COKFF algorithms without buffer...
buffer are just the same when all sensors are faultless. However, RMSEs of positions and velocities DFKFF and COKFF algorithms without buffer are almost the same, and both of them are seriously deflected when the second sensor is faulty at \(140 \leq k \leq 160\), as shown in Figs. 7 and 8. These simulation results illustrate that DFKFF and COKFF algorithms without buffer are equivalent, which do not have any fault-tolerance when any sensor is faulty. Meanwhile, they verify Theorem 1 as well.

### B. Example 2

Consider the following target tracking MUNS with two local sensors, where the system parameters in (1) are given by [12]:

\[
A = \begin{bmatrix} 0.95 & T \\ 0 & 0.95 \end{bmatrix}, \quad C_1 = [1, 1], \quad C_2 = [1, 1.5] \\
D_1 = D_2 = [1], \quad Q = I_2, \quad R_1 = 1, \quad R_2 = 1.5
\]

where \(T = 0.01\) is the sample period, two components of \(x_k\) is the position and velocity of the target.
The initial conditions are selected as $x_0 = [200, 10]^T$ and $P_0 = \text{diag}(0.5, 0.1)$. Following [29], in this paper it is assumed that $d_1 = d_2 = 3$ and $L_1 = L_2 = 5$. To exhibit the fault-tolerance of our proposed DFKFF algorithm with buffers, we assume that the second sensor is faulty and the measurement equation is given by $z^k_2 = C_2 x_k + D_2 v_2,k + f(k)$ where $f(k)$ satisfies $f(k) = 0.2k$ for $140 \leq k \leq 160$ and $f(k) = 0$ for $k < 140$ or $k > 160$. Fig. 9 shows traces of error covariances of LEs, DFKFF and COKFF algorithms with buffers. When all sensors are faultless, RMSEs of positions and velocities of DFKFF and COKFF algorithms with buffers are presented in Figs. 10 and 11, respectively. When the second sensor is faulty, RMSEs of positions and velocities of DFKFF and COKFF algorithms with buffers are depicted in Figs. 12 and 13, respectively.

Fig. 9 demonstrates that the trace of the error covariance of DFKFF algorithm with buffers is smaller than that of each LE, and little higher than that of COKFF. Figs. 10 and 11 show that RMSEs of positions and velocities of DFKFF algorithm with buffers are close to but little higher than that of COKFF algorithm with buffers when all sensors are faultless. The reason is that...
there is no reset during buffers. It can be seen from Figs. 12 and 13 that RMSEs of the position and velocity of COKFF algorithm with buffers diverges since the second sensor is faulty at \( 140 \leq k \leq 160 \). But RMSEs of the position and velocity of DFKFF algorithm with buffers can still remain smooth in a certain range, which explains that DFKFF algorithm with buffers can still track the position and velocity of the real signal. These simulation figures demonstrate that DFKFF algorithm with buffers has better fault-tolerance when the second sensor is faulty.

VI. CONCLUSION

In this paper, the problem of DFKFF has been investigated for a class of MUNSs with uncorrelated noises. We have presented the optimal DFKFF algorithm of MUNSs without buffer, and strictly proved that it is equivalent to COKFF algorithm of MUNSs without buffer. Based on the optimal local Kalman filter with a buffer of finite length for each subsystem and by use of finite length buffers to process measurement delay or loss, a sub-optimal DFKFF algorithm of MUNSs with buffers has been derived. Finally, simulation results have been summarized as follows: 1) DFKFF and COKFF algorithms of MUNSs without buffer are equivalent, which both do not have any fault-tolerance. 2) Our proposed suboptimal DFKFF algorithm of MUNSs with buffers has stronger fault-tolerance ability when any sensor is faulty, and its performance is close to that of COKFF algorithm of MUNSs with buffers when all sensors are faultless.

APPENDIX A

The Proof of Theorem 1: According to the matrix inversion lemma and the information form of Kalman filter, it is obtained that (12) and (13) are equivalent to

\[
\begin{align*}
\left(P_{k|k}^i\right)^{-1} &= \left(P_{k|k-1}^i\right)^{-1} + \left(\gamma_{i,k}^r C_{i,k}\right)^T \\
& \times \left(D_{i,k} R_{i,k} D_{i,k}^T\right)^{-1} \left(\gamma_{i,k}^r C_{i,k}\right)
\end{align*}
\]

(43)

Similarly, (24) and (25) are equivalent to

\[
\begin{align*}
P_{k|k}^{-1} &= P_{k|k-1}^{-1} + C_k^T \left(D_k R_k D_k^T\right)^{-1} C_k \\
P_{k|k}^{-1} \hat{x}_{k|k} &= P_{k|k-1}^{-1} \hat{x}_{k|k-1} + C_k^T \left(D_k R_k D_k^T\right)^{-1} z_k
\end{align*}
\]

(45) (46)

Firstly, \( P_{0|0}^f = P_{0|0}^i = P_0 \) and \( \hat{x}_{0|0}^f = \hat{x}_{0|0}^i = \hat{x}_0 \). Then, it follows from (7), (10), and (22) that:

\[
\sum_{i=1}^N \left(P_{1|0}^i\right)^{-1} = \sum_{i=1}^N \left(\tilde{A}_0 P_{0|0}^i A_0^T + Q_i^0\right)^{-1} = \sum_{i=1}^N \beta_i \left(\tilde{A}_0 P_{0|0}^i A_0^T + Q_i^0\right)^{-1} = \left(\sum_{i=1}^N \beta_i \right) \left(\tilde{A}_0 P_{0|0}^i A_0^T + Q_i^0\right)^{-1} = \left(\tilde{A}_0 P_{0|0}^i A_0^T + Q_i^0\right)^{-1} = P_{1|0}^i = P_{1|0}.
\]

(47)

Then substituting (7) into (10) yields

\[
P_{1|0}^i = A_0 P_{0|0}^i A_0^T + Q_0^i = \beta_i \left(\tilde{A}_0 P_{0|0}^i A_0^T + Q_i^0\right) = \beta_i \left(\tilde{A}_0 P_{0|0}^i A_0^T + Q_i^0\right) = \beta_i \left(P_{1|0}^i\right), \quad i = 0, 1, \ldots, N.
\]

According to (14), (43), and (45), we have

\[
\left(P_{1|1}^f\right)^{-1} = \sum_{i=1}^N \left(P_{1|1}^i\right)^{-1} = \sum_{i=1}^N \left[P_{1|0}^i + \left(\gamma_{1,i}^r C_{1,i}\right)^T \\
\times \left(D_{1,i} R_{1,i} D_{1,i}^T\right)^{-1} \left(\gamma_{1,k}^r C_{1,k}\right)\right]
\]

(51)

From the above formula, it is easy to get \( P_{1|1}^f = P_{1|1}^i \). It follows from (7) and \( \hat{x}_{0|0}^f = \hat{x}_{0|0}^i = \hat{x}_0 \) that:

\[
\hat{x}_{0|0}^i = \hat{x}_{0|0}.
\]

(49)

Then substituting (49) into (9) and (21) yields

\[
\hat{x}_{1|1}^i = \hat{x}_{1|1}.
\]

(50)

According to (15), (44), (46), (47), and (50), it can be obtained as follows:

\[
\left(P_{1|1}^i\right)^{-1} \hat{x}_{1|1}^i = \sum_{i=1}^N \left(P_{1|1}^i\right)^{-1} \hat{x}_{1|1}^i
\]

(51)

From (48) and (51), it is obvious to get \( \hat{x}_{1|1}^f = \hat{x}_{1|1} \).
Assume that $P_{k-1|k-1}^i = P_{k-1|k-1}$ and $\hat{x}_{k-1|k-1}^i = \hat{x}_{k-1|k-1}$. It follows from (7), (10), and (22) that:

$$\sum_{i = 1}^N \left( P_{k|k-1}^i \right)^{-1} = \sum_{i = 1}^N \left( A_{k-1} P_{k-1|k-1}^i A_{k-1}^T + Q_{k-1} \right)^{-1}$$

Then substituting (7) into (10) yields

$$P_{k|k-1}^i = A_{k-1} P_{k-1|k-1}^i A_{k-1}^T + Q_{k-1}$$

According to (14), (43), and (45), we have

$$\left( P_{k|k}^i \right)^{-1} = \sum_{i = 1}^N \left( P_{k|k-1}^i \right)^{-1}$$

From (15), (44), (46), (52), and (55), it follows that:

$$\left( P_{k|k}^i \right)^{-1} \hat{x}_{k|k}^i = \sum_{i = 1}^N \left( P_{k|k}^i \right)^{-1} \hat{x}_{k|k}^i$$

Then substituting (7) into (10) yields

$$P_{k|k-1}^i = A_{k-1} P_{k-1|k-1}^i A_{k-1}^T + Q_{k-1}$$

It follows from (7), (10), and (22) that:

$$\sum_{i = 1}^N \left( P_{k|k}^i \right)^{-1} \hat{x}_{k|k}^i = \sum_{i = 1}^N \left( P_{k|k}^i \right)^{-1} \hat{x}_{k|k}^i$$

From (53) and (56), it is obvious to get $\hat{x}_{k|k}^i = \hat{x}_{k|k}$. This completes proof.

The Proof of Theorem 2: According to the definition of $\kappa_i$ in (26), it is easily obtained that $\kappa_i$ is the earliest time when the measurement of the $i$th communication channel was updated at least, i.e., no new measurement $z_{t}^i$ ($t = k - L + 1, \ldots, k_i - 1, i = 1, 2, \ldots, N$) was received at time $t$. So, (36) and (37) can be easily deduced. Then, (38) and (39) follow directly from (27), (28), and (33).

It is easy to obtain $\hat{x}_{t|t-1,k}^i = x_t - \hat{x}_{t|t-1,k}$ from (34). According to Assumption 2.1, the $i$th one-step prediction error covariance $P_{t|t-1,k}^i$ is computed by

$$P_{t|t-1,k}^i = E \left\{ \left( \hat{x}_{t|t-1,k}^i - x_t \right) (\hat{x}_{t|t-1,k}^i - x_t)^T \right\}$$

From the above equation, it is easy to obtain $P_{k|k}^i = P_{k|k}$. It follows from (7) and $\hat{x}_{k-1|k-1}^i = \hat{x}_{k-1|k-1}$ that:

$$\hat{x}_{k-1|k-1}^i = \hat{x}_{k-1|k-1}$$

Then substituting (54) into (9) and (21) yields

$$\hat{x}_{k|k}^i = \hat{x}_{k|k}$$

Note that (57) yields (41).
It is worthy to note \( \dot{x}_{i}^{h,k} = x_{i} - \dot{x}_{i}^{h,k} \) from (34) and according to (28), \( \dot{x}_{i}^{h,k} \) is calculated by
\[
\dot{x}_{i}^{h,k} = x_{i} - \dot{x}_{i}^{h,k} - \gamma_{i}^{k} K_{i}^{k} (\hat{z}_{i}^{k} - C_{i} \dot{x}_{i}^{h,k})
\]
\[
= x_{i} - \dot{x}_{i}^{h,k} - \gamma_{i}^{k} K_{i}^{k} \left(C_{i}^{\text{t}} x_{i} + D_{i}^{\text{t}} v_{i,t} - C_{i} \dot{x}_{i}^{h,k}\right)
\]
\[
= (I - \gamma_{i}^{k} K_{i}^{k} C_{i}) \dot{x}_{i}^{h,k} - \gamma_{i}^{k} K_{i}^{k} D_{i}^{\text{t}} v_{i,t}
\]

Then, the local Kalman error covariance \( P_{i}^{k} \) is determined by
\[
P_{i}^{k} = E \left\{ \left( \dot{x}_{i}^{h,k} (\hat{z}_{i}^{k}) \right)^{T} \right\}
\]
\[
= \left( \left( I - \gamma_{i}^{k} K_{i}^{k} C_{i} \right) \dot{x}_{i}^{h,k} - \gamma_{i}^{k} K_{i}^{k} D_{i}^{\text{t}} v_{i,t} \right)^{T}
\]
\[
= \left( I - \gamma_{i}^{k} K_{i}^{k} C_{i} \right) P_{i}^{k} \left( I - \gamma_{i}^{k} K_{i}^{k} C_{i} \right)^{T}
\]
\[
+ \gamma_{i}^{k} K_{i}^{k} D_{i}^{\text{t}} R_{i,t} \left( \gamma_{i}^{k} K_{i}^{k} D_{i}^{\text{t}} \right)^{T}
\]
\[
= K_{i}^{k} \dot{x}_{i}^{h,k} + \gamma_{i}^{k} K_{i}^{k} C_{i} P_{i}^{k-1} \left( I - \gamma_{i}^{k} K_{i}^{k} C_{i} \right)
\]
\[
	imes \left( \gamma_{i}^{k} K_{i}^{k} C_{i}^{\text{T}} D_{i}^{\text{T}} R_{i,t} D_{i}^{\text{T}} \right)^{-1}
\]

Obtaining the following equation:

\[
-2 \gamma_{i}^{k} P_{i}^{k} + \gamma_{i}^{k} C_{i}^{\text{T}} D_{i}^{\text{T}} R_{i,t} D_{i}^{\text{T}} = 0
\]

where \( \gamma_{i}^{k} = 0 \) or 1. \( K_{i}^{k} \) can take any value if \( \gamma_{i}^{k} = 0 \) and \( K_{i}^{k} \) can be calculated by (59) if \( \gamma_{i}^{k} = 1 \). Therefore, the optimal local Kalman filter gain \( K_{i}^{k} \) is given by

\[
K_{i}^{k} = P_{i}^{k-1} \left( C_{i}^{\text{T}} D_{i}^{\text{T}} R_{i,t} D_{i}^{\text{T}} \right)^{-1}
\]

Then, substituting (60) into (58), the optimal local Kalman filter error covariance \( P_{i}^{k} \) is rewritten in the following form:

\[
P_{i}^{k} = P_{i}^{k-1} - \gamma_{i}^{k} K_{i}^{k} C_{i} P_{i}^{k-1}
\]

Note that (60) and (61) are just equations of (40) and (42), respectively. Repeating (36)–(42), i.e., the \( i \)th optimal local Kalman filter estimate \( \dot{x}_{i}^{k} \) and the error covariance \( P_{i}^{k} \) are determined by iterating \( k = k_{i} + 1 \) times from \( \dot{x}_{i}^{k-1} \) and \( P_{i}^{k-1} \) respectively. The proof is thus completed.

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